Eur. Phys. J. A **14**, 413–438 (2002)
DOI 10.1140/epja/i2001-10210-4

THE EUROPEAN PHYSICAL JOURNAL A

Some considerations on the restoration of Galilei invariance in the nuclear many-body problem

Part III: Energies of simple bound states

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Received: 20 February 2001
Communicated by D. Schwalm

Abstract. The effects of the restoration of Galilei invariance in the nuclear many-body problem on the energies of simple bound states are investigated. As examples we consider the oscillator ground states of ^{4}He, ^{16}O and ^{40}Ca as well as the various hole states with respect to these reference configurations. Density-independent as well as density-dependent interactions are studied. It turns out that the full restoration of Galilei-invariance yields considerable contributions on top of the trivial $1/A$ effect resulting from removing the center-of-momentum part of the Hamiltonian.

PACS. 21.60.-n Nuclear-structure models and methods

1 Introduction

This is the third paper in a series of six articles. In the first one [1] we have shown how Galilei-invariance can be restored in the nuclear many-body problem with the help of projection techniques. This paper was devoted to the development of the mathematical tools which are needed to project simple bound oscillator states into the center-of-momentum (COM) rest frame and to calculate the matrix elements of arbitrary operators in between such projected states. As a first application, furthermore in this paper the spectral functions and spectroscopic factors for the uncorrelated oscillator ground states of the three doubly even nuclei ^{4}He, ^{16}O and ^{40}Ca were investigated with and without projection into the COM rest frame. In agreement with earlier investigations [2] it turned out that the simple picture of an uncorrelated system is changed considerably if Galilei invariance is respected. For the deep-lying hole states we obtained a considerable depletion of the occupation due to the removal of the spurious admixtures resulting from the COM motion, which was compensated by an "over-occupation" of the (non-spurious) holes within the last occupied shell so that the sum rules for the total hole strengths are conserved. Similar effects were seen for the particle spectroscopic factors. In the second paper [3] then the effects of the restoration of Galilei invariance on the electromagnetic properties of simple bound states have been investigated. For this purpose, the charge and current form factors for elastic and inelastic electron scattering from the above ground states as well as in between the various one-hole states with respect to these reference configurations have been calculated. This has been done again in the "normal" way, in which the effects of the COM motion are approximated by the so-called "Tassie-Barker factor" [4], as well as with projection into the COM rest frame. Furthermore, in this paper the mathematical Coulomb sum rules and their first and second moments have been studied. For both, form factors as well as sum rules, again considerable effects due to the restoration of Galilei invariance were obtained. In the present article we shall investigate the energies of the oscillator ground-state configurations for the three considered doubly closed-shell nuclei as well as of the one-hole states with respect to them. Again we shall compare the results with and without the projection into the COM rest frame. Two types of Hamiltonians will be considered. The first is density independent and has the form

$$\hat{H}^{(1)}_{\rm int} \equiv \hat{T} - \hat{H}_{\rm com} + \hat{V}_{\rm Coul} + \hat{V}^{(1)}\,, \tag{1.1}$$

where

$$\hat{T} \equiv \sum_{i=1}^{A} \frac{\hat{p}^2(i)}{2M} \tag{1.2}$$

is the usual kinetic energy,

$$\hat{H}_{\rm com} \equiv \frac{\hat{P}_A^2}{2MA} \tag{1.3}$$

is the COM Hamiltonian, the subtraction of which is needed to ensure that (1.1) depends only on relative momenta,

$$\hat{V}_{\rm Coul} \equiv \frac{1}{2}\sum_{i\neq j=1}^{Z} \frac{e^2}{|\vec{r}_{ij}|} \tag{1.4}$$

is the Coulomb interaction between the protons of the considered system with the shorthand notation

$$r_{ij} \equiv |\vec{r}_{ij}| \equiv |\vec{r}_i - \vec{r}_j| \tag{1.5}$$

and, finally,

$$\hat{V}^{(1)} \equiv \frac{1}{2} \sum_{i \neq j=1}^{A} \left\{ \hat{V}_{\rm c}(\vec{r}_{ij}) + \hat{V}_{ls}(\vec{r}_{ij}) \right\} \tag{1.6}$$

is a density independent, effective nucleon-nucleon interaction. The first term is a central force

$$\hat{V}_{\rm c}(\vec{r}_{ij}) \equiv \sum_{S,\,T=0}^{1} u_{ST}\, \hat{P}_{ST}\, f_{ST}(r_{ij})\,, \tag{1.7}$$

where $\hat{P}_{ST}$ are projection operators onto the different spin-isospin channels, u_{ST} the strength parameters and $f_{ST}(r_{ij})$ the corresponding radial dependencies. The second term is a two-body spin-orbit force

$$\hat{V}_{ls}(\vec{r}_{ij}) \equiv \sum_{T=0}^{1} v_T\, \hat{P}_T\, g_T(r_{ij})\, \vec{l}_{ij} \cdot \vec{S}_{ij}\,, \tag{1.8}$$

which acts only in the spin $S = 1$ channels. Here $\vec{l}_{ij}$ is the operator of the relative orbital angular momentum of the two nucleons and $\vec{S}_{ij} = \vec{s}_i + \vec{s}_j$ the operator of their total spin. $\hat{P}_T$ projects onto the isospin channels and the corresponding strength parameters and radial dependencies have been denoted by v_T and $g_T(r_{ij})$, respectively. Tensor and quadratic spin-orbit forces could be added, but will not be considered in the present paper. The various terms of the Hamiltonian (1.1) will be studied in subsects. 2.1 to 2.5. The second Hamiltonian has, in addition to the form (1.1), a density-dependent term in the strong interaction

$$\hat{H}^{(2)}_{\rm int} \equiv \hat{H}^{(1)}_{\rm int} + \hat{V}^{\rm DD}_{\rm int} \tag{1.9}$$

where, for simplicity, we shall only consider a central interaction for the latter

$$\hat{V}^{\rm DD}_{\rm int} \equiv \frac{1}{2} \sum_{i \neq j=1}^{A} \sum_{S,\,T=0}^{1} w_{ST}\, \hat{P}_{ST}\, h^{\rm inv}_{ST}(\vec{r}_{ij}, \vec{R}_{ij} - \vec{R}_A)\,. \tag{1.10}$$

Here we have introduced

$$\vec{R}_{ij} \equiv \frac{1}{2} (\vec{r}_i + \vec{r}_j) \tag{1.11}$$

for the COM coordinate of the two nucleons. w_{ST} and $h^{\rm inv}_{ST}$ denote the strength-parameters and radial dependencies, respectively.

A dependence of the interaction on the COM coordinate of the two interacting nucleons is only possible in the nuclear medium, where the remaining $A-2$ nucleons can compensate the correponding momentum transfer so that the total linear momentum of the system remains zero. As we shall see, this is ensured by writing eq. (1.10) in relative coordinates $(\vec{r}_i - \vec{R}_A + \vec{r}_j - \vec{R}_A)/2$, which, unfortunately, makes out of eq. (1.10) an A-body force. Normally eq. (1.10) is not written in this Galilei-invariant form but instead of $h^{\rm inv}$ one uses $h^{\rm nor}(r_{ij}, R_{ij})$ for the radial dependence. This approximation is made, *e.g.*, in the normal G-matrix calculations [5] but also for more phenomenological forces like the Gogny interaction [6] in which the density-dependent term has the simple form

$$\hat{V}^{\rm DD}_{\rm Gogny,\,nor}(1,2) = t_0(1+x_0\hat{P}_\sigma)\delta^{(3)}(\vec{r}_{12})\,\rho_0^\alpha(\vec{R}_{12})\,, \tag{1.12}$$

where $\hat{P}_\sigma$ is the spin exchange operator and $\rho(\vec{R}_{12})$ the density of the system at the COM coordinate of the two interacting nucleons. The subscript "0" indicates that only the scalar (monopole) part of this density should be used in order to guarantee the rotational invariance of (1.11). In the Gogny force (1.12) one usually takes $x_0 = 1$ and $\alpha = 1/3$ so that one is left with only one free parameter t_0. Density dependent forces of the type (1.10) and (1.12) (as well as its Galilei-invariant form) will be studied in sect. 3.

2 Energies with density-independent interactions

In this section the contributions of the various terms of the Hamiltonian (1.1) will be calculated with and without projection into the COM rest frame. The mathematical tools for this have been developed in [1]. Furthermore, in ref. [3] the expressions for the matrix elements of general one-body operators within our simple oscillator configurations have been given. The corresponding formulas (with and without projection into the COM rest frame) hence will not be repeated in the present article.

2.1 The kinetic energy and the COM Hamiltonian

We start with the kinetic energy. In momentum space representation the corresponding operator has the form

$$\hat{T} = \frac{\hbar\omega}{2} \int d^3\vec{k}_1\, (b\vec{k}_1)^2 \sum_1 c^\dagger_{\vec{k}_1\,1} c_{\vec{k}_1\,1}\,, \tag{2.1}$$

where b is the oscillator length parameter. Obviously, this operator depends via

$$\hat{T} = \frac{1}{2M} \sum_{i=1}^{A} \hat{p}^2(i) = \frac{1}{2M} \sum_{i=1}^{A} \left(\hat{p}(i) - \frac{1}{A}\hat{P}_A^2 \right)^2 + \frac{\hat{P}_A^2}{2MA} \tag{2.2}$$

not only on the relative momenta but also on the total momentum of the system. This fact does not matter, if we project into the COM rest frame (since there $\vec{P}_A = 0$), but does effect the unprojected matrix elements. In order to compensate the corresponding trivial $1/A$ effect, we have to subtract the COM term as done in eq. (1.1). Let us start, however, with the unmodified operator (2.1).

Using (2.5) out of ref. [3] with $\vec{\lambda} = 0$ and (2.2) out of the same article, we immediately obtain for the expectation values of the kinetic energy within the unprojected ground-state configurations of the three considered doubly closed-shell nuclei

$$\langle\,|\hat{T}|\,\rangle = \hbar\omega \left\{ \begin{array}{lll} 3, & \text{for} & {}^{4}\text{He} \\ 18, & \text{for} & {}^{16}\text{O} \\ 60, & \text{for} & {}^{40}\text{Ca} \end{array} \right\}, \tag{2.3}$$

a result which obviously could have been obtained by simply counting the oscillator quanta in the different occupied major shells, too. The only non-vanishing matrix elements between unprojected one-hole states can be obtained from eq. (2.6) in [3] (or again by simple counting). Here one gets

$$\langle\,|b^{\dagger}_{Hh}\hat{T}\,b_{H'h'}|\,\rangle = \Delta_{h'h}\hbar\omega \times \left\{ \begin{array}{lll} 3-\frac{3}{4}, & \text{for } H=H'=0s & \text{in } {}^{4}\text{He} \\ 18-\frac{3}{4}, & \text{for } H=H'=0s & \text{in } {}^{16}\text{O} \\ 18-\frac{5}{4}, & \text{for } H=H'=0p & \text{in } {}^{16}\text{O} \\ 60-\frac{3}{4}, & \text{for } H=H'=0s & \text{in } {}^{40}\text{Ca} \\ 60-\frac{5}{4}, & \text{for } H=H'=0p & \text{in } {}^{40}\text{Ca} \\ 60-\frac{7}{4}, & \text{for } H=H'=0d & \text{in } {}^{40}\text{Ca} \\ 60-\frac{7}{4}, & \text{for } H=H'=1s & \text{in } {}^{40}\text{Ca} \\ \frac{1}{2}\sqrt{\frac{3}{2}}, & \text{for } H=0s,\ H'=1s & \text{in } {}^{40}\text{Ca} \end{array} \right\}. \tag{2.4}$$

On the other hand, we obtain for the expectation values of the kinetic-energy operator within the projected ground-state configurations of the considered doubly closed-shell nuclei, evaluating (2.7) out of [3],

$$\frac{\langle\,|\hat{T}\hat{C}_A(0)|\,\rangle}{\langle\,|\hat{C}_A(0)|\,\rangle} = \hbar\omega \left\{ \begin{array}{lll} 3-\frac{3}{4}, & \text{for} & {}^{4}\text{He} \\ 18-\frac{3}{4}, & \text{for} & {}^{16}\text{O} \\ 60-\frac{3}{4}, & \text{for} & {}^{40}\text{Ca} \end{array} \right\}. \tag{2.5}$$

Comparing these with (2.3) and using (2.17) of ref. [1] one sees immediately that for the non-spurious oscillator configurations $|\,\rangle$

$$\frac{\langle\,|\hat{T}\hat{C}_A(0)|\,\rangle}{\langle\,|\hat{C}_A(0)|\,\rangle} = \frac{\left\langle\,\left|\left[\hat{T} - \frac{\hat{P}^2_A}{2MA}\right]\hat{C}_A(0)\right|\,\right\rangle}{\langle\,|\hat{C}_A(0)|\,\rangle} = \left\langle\,\left|\left[\hat{T} - \frac{\hat{P}^2_A}{2MA}\right]\right|\,\right\rangle \tag{2.6}$$

with the first equal sign due to the fact that we project into the COM rest frame, in which the nucleus has vanishing total linear momentum. Evaluating (2.11) out of [3] for the projected hole-hole matrix elements and normalising them according to eqs. (2.63) and (2.64) out of ref. [1] one obtains

$$\langle (Hh)^{-1},(0)|\hat{T}|(H'h')^{-1},(0)\rangle = \Delta_{h'h}\hbar\omega \cdot \left\{ \begin{array}{lll} 3-\frac{3}{4}-\frac{3}{4}, & \text{for } H=H'=0s & \text{in } {}^{4}\text{He} \\ 18-\frac{3}{4}-\frac{3}{4}, & \text{for } H=H'=0s & \text{in } {}^{16}\text{O} \\ 18-\frac{5}{4}-\frac{3}{4}, & \text{for } H=H'=0p & \text{in } {}^{16}\text{O} \\ 60-\frac{3}{4}-\frac{3}{4}, & \text{for } H=H'=0\tilde{s} & \text{in } {}^{40}\text{Ca} \\ 60-\frac{5}{4}-\frac{3}{4}, & \text{for } H=H'=0p & \text{in } {}^{40}\text{Ca} \\ 60-\frac{7}{4}-\frac{3}{4}, & \text{for } H=H'=0d & \text{in } {}^{40}\text{Ca} \\ 60-\frac{7}{4}-\frac{3}{4}, & \text{for } H=H'=1s & \text{in } {}^{40}\text{Ca} \\ 2\sqrt{\frac{5}{47}}, & \text{for } H=0\tilde{s},\ H'=1s & \text{in } {}^{40}\text{Ca} \end{array} \right\}. \tag{2.7}$$

Comparing these with (2.4), we see that for both holes beeing "non-spurious", *i.e.*, from the last occupied major shell, we have again

$$\begin{aligned} &\langle (Hh)^{-1},(0)|\hat{T}|(Hh)^{-1},(0)\rangle = \\ &\langle (Hh)^{-1},(0)|\left[\hat{T} - \frac{\hat{P}^2_{A-1}}{2M(A-1)}\right]|(Hh)^{-1},(0)\rangle = \\ &\left\langle\,\left|b^{\dagger}_{Hh}\left[\hat{T} - \frac{\hat{P}^2_{A-1}}{2M(A-1)}\right]b_{Hh}\right|\,\right\rangle, \end{aligned} \tag{2.8}$$

where use has been made of the fact that there are no non-diagonal matrix elements in this case.

It remains to evaluate the COM Hamiltonian in the case of hole states with excitation energies $\geq 1\hbar\omega$. For this purpose we write the two-body part of the operator

$$\begin{aligned} \frac{\hat{P}^2_{A-1}}{2M(A-1)} &= \frac{1}{2M(A-1)}\left(\sum_{i=1}^{A-1}\hat{p}(i)\right)^2 = \\ &\frac{1}{2M(A-1)}\sum_{i=1}^{A-1}\hat{p}^2(i) + \frac{1}{2M(A-1)}\sum_{i\neq j=1}^{A-1}\hat{p}(i)\cdot\hat{p}(j) = \\ &\frac{1}{A-1}\hat{T} + \frac{1}{A-1}\hat{R} \end{aligned} \tag{2.9}$$

in momentum space representation

$$\hat{R} = \frac{\hbar\omega}{2}\sum_{12}\int \mathrm{d}^3\vec{k}_1 \int \mathrm{d}^3\vec{k}_2\ \vec{\kappa}_1\cdot\vec{\kappa}_2\ c^{\dagger}_{\vec{k}_1 1}c^{\dagger}_{\vec{k}_2 2}c_{\vec{k}_2 2}c_{\vec{k}_1 1} \tag{2.10}$$

with our usual convention $\vec{\kappa}_i \equiv b\vec{k}_i$ for $i = 1,2$. The matrix elements of (2.10) within unprojected one-hole states have then the form

$$\langle\, |b^\dagger_{Hh}\hat{R}b_{H'h'}|\,\rangle \;=\; \frac{\hbar\omega}{2}\frac{1}{\pi\sqrt{\pi}}\int \mathrm{d}^3\vec{\kappa}_1\,\exp\{-\kappa_1^2\}\frac{1}{\pi\sqrt{\pi}} \times\int \mathrm{d}^3\vec{\kappa}_2\,\exp\{-\kappa_2^2\}\,\vec{\kappa}_1\cdot\vec{\kappa}_2 \cdot\varDelta_{h'h}\Big[\delta_{H'H}\{16y(\vec{\kappa}_1,\,\vec{\kappa}_1)\,y(\vec{\kappa}_2,\,\vec{\kappa}_2) - 4y(\vec{\kappa}_2,\,\vec{\kappa}_1)\,y(\vec{\kappa}_1,\,\vec{\kappa}_2)\} - (\vec{\kappa}_1|H)\{4(H'|\vec{\kappa}_1)y(\vec{\kappa}_2,\,\vec{\kappa}_2) - (H'|\vec{\kappa}_2)y(\vec{\kappa}_2,\,\vec{\kappa}_1)\} + (\vec{\kappa}_2|H)\{(H'|\vec{\kappa}_1)y(\vec{\kappa}_1,\,\vec{\kappa}_2) - 4(H'|\vec{\kappa}_2)y(\vec{\kappa}_1,\,\vec{\kappa}_1)\}\Big]\,, \tag{2.11}$$

which can be evaluated easily with the help of eqs. (2.9) and (2.26) out of ref. [1]. Using furthermore (2.4) we obtain for the non-vanishing matrix elements

$$\left\langle\left|b^\dagger_{Hh}\left[\hat{T}-\frac{\hat{P}^2_{A-1}}{2M(A-1)}\right]b_{H'h'}\right|\right\rangle = \varDelta_{h'h}\,\hbar\omega \cdot\left\{\begin{array}{lll} 3-\frac{3}{4}-\frac{3}{4}, & \text{for } H=H'=0s & \text{in } ^4\text{He}\\ 18-\frac{3}{4}-\frac{3}{4}\left(\frac{17}{15}\right), & \text{for } H=H'=0s & \text{in } ^{16}\text{O}\\ 18-\frac{5}{4}-\frac{3}{4}, & \text{for } H=H'=0p & \text{in } ^{16}\text{O}\\ 60-\frac{3}{4}-\frac{3}{4}\left(\frac{81}{78}\right), & \text{for } H=H'=0s & \text{in } ^{40}\text{Ca}\\ 60-\frac{5}{4}-\frac{3}{4}\left(\frac{41}{39}\right), & \text{for } H=H'=0p & \text{in } ^{40}\text{Ca}\\ 60-\frac{7}{4}-\frac{3}{4}, & \text{for } H=H'=0d & \text{in } ^{40}\text{Ca}\\ 60-\frac{7}{4}-\frac{3}{4}, & \text{for } H=H'=1s & \text{in } ^{40}\text{Ca}\\ 2\sqrt{\frac{5}{47}}\left(\sqrt{\frac{470}{507}}\right), & \text{for } H=0s,\ H'=1s & \text{in } ^{40}\text{Ca} \end{array}\right\}. \tag{2.12}$$

This has now to be compared to the projected results (2.7). As can be seen, for the hole states with excitation energies $\geq 1\hbar\omega$ the projected kinetic energies for $A-1$ systems (2.7) are slightly larger than their approximate counterparts (2.12). Assuming typical oscillator lengths of $b = 1.8$ fm for ^{16}O and $b = 2.0$ fm for ^{40}Ca, one obtains for the $0s$-hole in ^{16}O $+\,1.28$ MeV, for the $0p$- and $0\tilde{s}$-hole in ^{40}Ca $+\,0.40$ MeV and $+\,0.30$ MeV, respectively, and for the projected non-diagonal $0\tilde{s}$-$1s$ matrix element in ^{40}Ca $+\,0.25$ MeV more than in the approximate calculation.

2.2 Density-independent two-body forces

We shall now consider density-independent two-body forces of the type used in (1.1). Their general structure in momentum space representation is

$$\hat{V} = \frac{1}{4}\sum_{1234}\int \mathrm{d}^3\vec{k}_1\int \mathrm{d}^3\vec{k}_2\int \mathrm{d}^3\vec{k}_3\int \mathrm{d}^3\vec{k}_4 \cdot\Big\{\langle\vec{k}_1\,1\,\vec{k}_2\,2\,|\hat{V}(1,2)|\vec{k}_3\,3\,\vec{k}_4\,4\,\rangle - \langle\vec{k}_1\,1\,\vec{k}_2\,2\,|\hat{V}(1,2)|\vec{k}_4\,4\,\vec{k}_3\,3\,\rangle\Big\} \cdot c^\dagger_{\vec{k}_1\,1}\,c^\dagger_{\vec{k}_2\,2}\,c_{\vec{k}_4\,4}\,c_{\vec{k}_3\,3}\,, \tag{2.13}$$

where the sum runs over the spin and isospin quantum numbers of the plane-wave states. Let us first consider the radial matrix element only. Spin and isospin dependencies will be added later. If the radial dependence of the force is of the form $f(r_{12})$, then we obtain for the direct term

$$\langle\vec{k}_1\vec{k}_2|f(r_{12})|\vec{k}_3\vec{k}_4\rangle = (2\pi)^{-6}\int \mathrm{d}^3\vec{r}_1\int \mathrm{d}^3\vec{r}_2 \times\exp\{-i\vec{k}_1\cdot\vec{r}_1 - i\vec{k}_2\cdot\vec{r}_2\} \cdot f(r_{12})\exp\{i\vec{k}_3\cdot\vec{r}_1 + i\vec{k}_4\cdot\vec{r}_2\} = (2\pi)^{-6}\int \mathrm{d}^3\vec{R}_{12}\,\exp\{i(\vec{k}_3+\vec{k}_4-\vec{k}_1-\vec{k}_2)\cdot\vec{R}_{12}\} \cdot\int \mathrm{d}^3\vec{r}_{12}\,\exp\left\{-i\frac{1}{2}(\vec{k}_1-\vec{k}_2)\cdot\vec{r}_{12}\right\} \times f(r_{12})\exp\left\{i\frac{1}{2}(\vec{k}_3-\vec{k}_4)\cdot\vec{r}_{12}\right\} = \delta^{(3)}(\vec{Q}'-\vec{Q})\,(2\pi)^{-3} \times\int \mathrm{d}^3\vec{r}_{12}\,\exp\{-i\vec{q}\cdot\vec{r}_{12}\}f(r_{12})\exp\{i\vec{q}\,'\cdot\vec{r}_{12}\} \equiv (2\pi)^{-3/2}\delta^{(3)}(\vec{Q}\,'-\vec{Q})\,F[\vec{q}-\vec{q}\,']\,, \tag{2.14}$$

where we have used the usual transformation

$$\vec{r}_{12} \equiv \vec{r}_1 - \vec{r}_2\,, \qquad \vec{R}_{12} \equiv \frac{1}{2}(\vec{r}_1 + \vec{r}_2)\,, \qquad \vec{r}_1 \equiv \vec{R}_{12} + \vec{r}_{12}/2\,, \qquad \vec{r}_2 \equiv \vec{R}_{12} - \vec{r}_{12}/2\,, \tag{2.15}$$

for the relative and the COM coordinate of the two nucleons. Furthermore, we have introduced

$$\begin{array}{ll} \vec{q} \equiv \frac{1}{2}(\vec{k}_1 - \vec{k}_2)\,, & \vec{Q} \equiv \vec{k}_1 + \vec{k}_2\,,\\ \vec{q}\,' \equiv \frac{1}{2}(\vec{k}_3 - \vec{k}_4)\,, & \vec{Q}\,' \equiv \vec{k}_3 + \vec{k}_4\,,\\ \vec{k}_1 \equiv \vec{Q}/2 + \vec{q}, & \vec{k}_2 \equiv \vec{Q}/2 - \vec{q},\\ \vec{k}_3 \equiv \vec{Q}\,'/2 + \vec{q}\,'\,, & \vec{k}_4 \equiv \vec{Q}\,'/2 - \vec{q}\,'\,, \end{array} \tag{2.16}$$

for the relative and total momenta of the two nucleons after and before the interaction and used the abbreviation $F[\vec{q}-\vec{q}\,']$ for the Fourier transform of the radial dependence $f(r_{12})$. In the same way, we get for the exchange term

$$\langle\vec{k}_1\vec{k}_2|f(r_{12})|\vec{k}_4\vec{k}_3\rangle = \delta^{(3)}(\vec{Q}\,'-\vec{Q})\,(2\pi)^{-3} \cdot\int \mathrm{d}^3\vec{r}_{12}\,\exp\{-i\vec{q}\cdot\vec{r}_{12}\}f(r_{12})\exp\{-i\vec{q}\,'\cdot\vec{r}_{12}\} \equiv (2\pi)^{-3/2}\delta^{(3)}(\vec{Q}\,'-\vec{Q})\,F[\vec{q}+\vec{q}\,']\,. \tag{2.17}$$

We now define

$$\vec{k} \equiv \tfrac{1}{\sqrt{2}}\,(\vec{q}-\vec{q}\,')\,, \qquad \vec{K} \equiv \tfrac{1}{\sqrt{2}}\,(\vec{q}+\vec{q}\,')\,, \qquad \vec{q} \equiv \tfrac{1}{\sqrt{2}}\left(\vec{K}+\vec{k}\right)\,, \qquad \vec{q}\,' \equiv \tfrac{1}{\sqrt{2}}\left(\vec{K}-\vec{k}\right) \tag{2.18}$$

and

$$\vec{T} \equiv \frac{1}{\sqrt{2}}\vec{Q}\,. \tag{2.19}$$

$$\frac{\int \mathrm{d}^3\vec{T}\,\langle\,|c^\dagger_{(\vec{T}+\vec{K}+\vec{k})/\sqrt{2}\,1}\,c^\dagger_{(\vec{T}-\vec{K}-\vec{k})/\sqrt{2}\,2}\,c_{(\vec{T}-\vec{K}+\vec{k})/\sqrt{2}\,4}\,c_{(\vec{T}+\vec{K}-\vec{k})/\sqrt{2}\,3}\,\hat{C}_A(0)|\,\rangle^{(\mathrm{a})}}{\langle\,|\hat{C}_A(0)|\,\rangle} =$$
$$\int \mathrm{d}^3\vec{T}\,\langle\,|c^\dagger_{(\vec{T}+\vec{K}+\vec{k})/\sqrt{2}\,1}\,c^\dagger_{(\vec{T}-\vec{K}-\vec{k})/\sqrt{2}\,2}\,c_{(\vec{T}-\vec{K}+\vec{k})/\sqrt{2}\,4}\,c_{(\vec{T}+\vec{K}-\vec{k})/\sqrt{2}\,3}|\,\rangle^{(\mathrm{a})}\,. \tag{2.26}$$

Using this notation for a central interaction of the type (1.7), the expression (2.13) can be written as

$$\hat{V}_\mathrm{c} = \frac{1}{4}\sum_{1234}\frac{1}{\pi\sqrt{\pi}}\int \mathrm{d}^3\vec{k}\int \mathrm{d}^3\vec{K} \cdot \sum_{ST}\Big\{U^{ST}_{1234}\,F_{ST}[\sqrt{2}\vec{k}] - U^{ST}_{1243}\,F_{ST}[\sqrt{2}\vec{K}]\Big\} \cdot \int \mathrm{d}^3\vec{T}\,c^\dagger_{(\vec{T}+\vec{K}+\vec{k})/\sqrt{2}\,1}\,c^\dagger_{(\vec{T}-\vec{K}-\vec{k})/\sqrt{2}\,2} \cdot c_{(\vec{T}-\vec{K}+\vec{k})/\sqrt{2}\,4}\,c_{(\vec{T}+\vec{K}-\vec{k})/\sqrt{2}\,3}\,, \tag{2.20}$$

where we have introduced

$$U^{ST}_{ijrs} \equiv \langle ij|u_{ST}\hat{P}_{ST}|rs\rangle \tag{2.21}$$

for the spin-isospin matrix element and $F_{ST}[\vec{p}]$ for the Fourier transform of the radial function $f_{ST}(r_{12})$ at momentum $\vec{p}$. As we shall see in subsect. 2.3, the Coulomb interaction (1.4) can be written in a similar form.

The two-body spin-orbit force (1.8) can be treated using the same notation. Here we obtain

$$\hat{V}_{ls} = \frac{1}{4}\sum_{1234}\frac{1}{\pi\sqrt{\pi}}\int \mathrm{d}^3\vec{k}\int \mathrm{d}^3\vec{K}\sum_{T,\,\mu}(-)^\mu \cdot \Big\{V^{T,\,\mu}_{1234}\,G^T_{-\mu}[\sqrt{2}\vec{k},\,\vec{K}-\vec{k}] + V^{T,\,\mu}_{1243}\,G^T_{-\mu}[\sqrt{2}\vec{K},\,\vec{K}-\vec{k}]\Big\} \cdot \int \mathrm{d}^3\vec{T}\,c^\dagger_{(\vec{T}+\vec{K}+\vec{k})/\sqrt{2}\,1}\,c^\dagger_{(\vec{T}-\vec{K}-\vec{k})/\sqrt{2}\,2} \cdot c_{(\vec{T}-\vec{K}+\vec{k})/\sqrt{2}\,4}\,c_{(\vec{T}+\vec{K}-\vec{k})/\sqrt{2}\,3}\,, \tag{2.22}$$

where now

$$V^{T,\,\mu}_{ijrs} \equiv \langle ij|v_{ST}\hat{P}_T S_\mu|rs\rangle \tag{2.23}$$

for the spin-isospin matrix element and

$$G^T_{-\mu}[\sqrt{2}\vec{p},\vec{K}-\vec{k}] \equiv (2\pi)^{-3/2}\int \mathrm{d}^3\vec{r}_{12}\,\exp\{-i\sqrt{2}\vec{p}\cdot\vec{r}_{12}\} \cdot \frac{1}{\sqrt{2}}\left(\vec{r}_{12}\times(\vec{K}-\vec{k})\right)_{-\mu} g_T(r_{12}) \tag{2.24}$$

for $\vec{p}$ equal to $\vec{k}$ for the direct and equal to $\vec{K}$ for the exchange term. The sum over μ takes care of the vector character in spin and ordinary space. Here the spherical representation ($\mu = -1, 0, 1$) has been used. Note, that neither (2.20) nor (2.22) changes the total linear momentum of the system.

For the oscillator ground-state configurations of the considered doubly closed-shell nuclei, these expressions can be easily evaluated. In the unprojected case one gets

$$\int \mathrm{d}^3\vec{T}\,\langle\,|c^\dagger_{(\vec{T}+\vec{K}+\vec{k})/\sqrt{2}\,1}\,c^\dagger_{(\vec{T}-\vec{K}-\vec{k})/\sqrt{2}\,2} \cdot c_{(\vec{T}-\vec{K}+\vec{k})/\sqrt{2}\,4}\,c_{(\vec{T}+\vec{K}-\vec{k})/\sqrt{2}\,3}|\,\rangle^{(\mathrm{a})} =$$
$$\frac{b^3}{\pi\sqrt{\pi}}\exp\{-D^2-d^2\}\frac{1}{\pi\sqrt{\pi}}\int \mathrm{d}^3\vec{\Lambda}\exp\{-\Lambda^2\} \cdot 2\Delta_{42}\Delta_{31}\,y\left((\vec{\Lambda}-\vec{D}+\vec{d})/\sqrt{2},\,(\vec{\Lambda}-\vec{D}-\vec{d})/\sqrt{2}\right) \cdot y\left(\frac{1}{\sqrt{2}}(\vec{\Lambda}+\vec{D}-\vec{d}),\,\frac{1}{\sqrt{2}}(\vec{\Lambda}+\vec{D}+\vec{d})\right) =$$
$$\frac{b^3}{\pi\sqrt{\pi}}2\Delta_{42}\Delta_{31}\exp\{-D^2-d^2\} \cdot \left\{\begin{array}{ll} 1\,, & \text{for } {}^4\mathrm{He} \\ \frac{31}{4} - 5\,d^2 + d^4 + 3\,D^2 - 2\,d^2D^2 + D^4\,, & \text{for } {}^{16}\mathrm{O} \\ \frac{1945}{64} - \frac{305}{8}\,d^2 + \frac{145}{8}\,d^4 - \frac{7}{2}\,d^6 + \frac{1}{4}\,d^8 & \\ +\frac{135}{8}\,D^2 - \frac{85}{4}\,d^2D^2 + \frac{15}{2}\,d^4D^2 - d^6D^2 & \\ +\frac{49}{8}\,D^4 - \frac{9}{2}\,d^2D^4 + \frac{3}{2}\,d^4D^4 & \\ \quad +\frac{1}{2}\,D^6 - d^2D^6 + \frac{1}{4}\,D^8\,, & \text{for } {}^{40}\mathrm{Ca} \end{array}\right\}, \tag{2.25}$$

where we have introduced $\vec{\Lambda} \equiv b\vec{T}$, $\vec{d} \equiv b\vec{k}$ and $\vec{D} \equiv b\vec{K}$ and applied eqs. (2.25) and (2.26) out of ref. [1]. Finally, we have made use of the fact that later on (2.25) will be multiplied by an anti-symmetrized matrix element. This is indicated by the superscript "(a)".

Since the $|\,\rangle$ are all "non-spurious" oscillator configurations and since the interaction depends only on relative coordinates we immediately obtain for the correpsonding COM-projected matrix elements

see eq. (2.26) *above*

This expression can be easily checked explicitly.

We come now to the hole-hole matrix elements. In the unprojected case we have here

$$\int \mathrm{d}^3\vec{T}\,\langle\,|b^\dagger_{Hh}c^\dagger_{(\vec{T}+\vec{K}+\vec{k})/\sqrt{2}\,1}\,c^\dagger_{(\vec{T}-\vec{K}-\vec{k})/\sqrt{2}\,2} \cdot c_{(\vec{T}-\vec{K}+\vec{k})/\sqrt{2}\,4}\,c_{(\vec{T}+\vec{K}-\vec{k})/\sqrt{2}\,3}b_{H'h'}\,|\,\rangle^{(\mathrm{a})} = \tag{2.27}$$

$$\frac{b^3}{\pi\sqrt{\pi}}\,\exp\{-D^2-d^2\}\frac{1}{\pi\sqrt{\pi}}\int \mathrm{d}^3\vec{\Lambda}\,\exp\{-\Lambda^2\}$$
$$\cdot 2\Delta_{42}\,y\left((\vec{\Lambda}-\vec{D}+\vec{d})/\sqrt{2},\,(\vec{\Lambda}-\vec{D}-\vec{d})/\sqrt{2}\right)$$
$$\cdot\Big\{\Delta_{h'h}\delta_{H'H}\Delta_{31}\,y\left((\vec{\Lambda}+\vec{D}-\vec{d})/\sqrt{2},\,(\vec{\Lambda}+\vec{D}+\vec{d})/\sqrt{2}\right)$$
$$-2\Delta_{1h'}\Delta_{3h}\left((\vec{\Lambda}+\vec{D}-\vec{d})/\sqrt{2}|H\right)\left(H'|(\vec{\Lambda}+\vec{D}+\vec{d})/\sqrt{2}\right)\Big\}, \quad (2.27)$$

where we have used eqs. (2.23) to (2.25) from ref. [1]. Again this expression can be evaluated easily with the help of (2.26) and (2.9) out of the same paper. In order to avoid errors, however, here the use of some computer algebra program is advisable. For the corresponding (not yet normalized) COM-projected matrix element one obtains

$$\int \mathrm{d}^3\vec{T}\,\langle\,|b^\dagger_{Hh}c^\dagger_{(\vec{T}+\vec{K}+\vec{k})/\sqrt{2}\,1}\,c^\dagger_{(\vec{T}-\vec{K}-\vec{k})/\sqrt{2}\,2}$$
$$\cdot\, c_{(\vec{T}-\vec{K}+\vec{k})/\sqrt{2}\,4}\,c_{(\vec{T}+\vec{K}-\vec{k})/\sqrt{2}\,3}\hat{C}_{A-1}(0)b_{H'h'}\,|\,\rangle^{(\mathrm{a})} =$$
$$\left(\frac{4}{A-1}\right)^{3/2} b^3\pi\sqrt{\pi}\,\frac{b^3}{\pi\sqrt{\pi}}\,\exp\{-D^2-d^2\}\frac{1}{\pi\sqrt{\pi}}$$
$$\cdot\int\mathrm{d}^3\vec{u}\,\exp\{-u^2\}\frac{1}{\pi\sqrt{\pi}}\int\mathrm{d}^3\vec{v}\,\exp\{-v^2\}\,2\Delta_{42}\,x(\vec{\beta}_4',\vec{\beta}_2)$$
$$\cdot\Big\{\Delta_{h'h}\Delta_{31}\,z^{-1}_{H'H}(\vec{\beta},\vec{\beta}\,')\,x(\vec{\beta}_3\,',\vec{\beta}_1)$$
$$-2\Delta_{1h'}\Delta_{3h}\,r_H(\vec{\beta}_3\,',\vec{\beta})\,\tilde{r}_{H'}(\vec{\beta}_1,\vec{\beta}\,')\Big\}, \quad (2.28)$$

where we have used the operator (2.27) from ref. [1] with $\vec{q}_1 = \vec{q}_2 = 0$ and eqs. (2.34) as well as (2.36) to (2.38) of ref. [1] for the elementary contractions. Furthermore,

$$\vec{\beta} \equiv \vec{\beta}\,' \equiv \sqrt{\frac{2}{A-1}}\,\vec{u},$$
$$\vec{\beta}_1 \equiv -i\vec{v}+i(\vec{D}+\vec{d}), \qquad \vec{\beta}_2 \equiv -i\vec{v}-i(\vec{D}+\vec{d}),$$
$$\vec{\beta}_3\,' \equiv -i\vec{v}+i(\vec{D}-\vec{d}), \qquad \vec{\beta}_4\,' \equiv -i\vec{v}-i(\vec{D}-\vec{d}). \quad (2.29)$$

With the help of the $z^{-1}_{H'H}$ out of eqs. (2.43) to (2.45) from ref. [1], $\tilde{r}$, r and x given by eqs. (2.47) to (2.53) from [1] and a computer algebra program, the evaluation of (2.28) and its subsequent normalisation via eqs. (2.63) and (2.64) out of [1] is now straightforward.

Before doing so, however, it is instructive to discuss some general properties of (2.28). As can be seen easily from (2.29), in the first term of (2.28), the $z^{-1}_{H'H}$ depend only on $\vec{u}$, while the functions x depend only on $\vec{v}$. Thus, the two integrations can be performed for this term separately. The integration over $\vec{u}$ gives just the overlap matrix discussed in subsect. 2.4 of ref. [1] and thus drops out, if (2.28) is normalized according to eqs. (2.63) and (2.64) of [1]. The integral over $\vec{v}$ on the other hand, gives for this term just the same result as (2.25). For the second term the situation is different. Here, the two integration factor only, if the two holes are out of the last occupied major shell, since then, again $\tilde{r}$ and r depend only on $\vec{v}$. Using the normalisation (2.63) out of [1], one sees immediately that for non-spurious hole-states the normalized version of (2.28) reduces to the unprojected result (2.27). This is not the case for the hole-states with excitation energies $\geq 1\hbar\omega$. We shall evaluate the above formulas explicitly for the case of the Coulomb interaction and the case of Gaussian-shaped central and spin-orbit interactions in the next two subsections.

2.3 The Coulomb energy

Let us start with the Coulomb interaction (1.4) which acts only between protons and does not depend on the spin of the two-nucleon states. The radial dependence is here $f_{\mathrm{Coul}}(r_{12}) = 1/r_{12}$. Consequently, the corresponding Fourier transform is

$$F[\sqrt{2}\vec{p}] \equiv (2\pi)^{-3/2}\int\mathrm{d}^3\vec{r}_{12}\,\exp\{-i\sqrt{2}\vec{p}\cdot\vec{r}_{12}\}\,\frac{1}{r_{12}} =$$
$$\frac{b^2}{4\pi\sqrt{2\pi}}\lim_{\alpha\to 0}\int\mathrm{d}^3\vec{y}\,\exp\{-ib\vec{p}\cdot\vec{y}\}\,\frac{1}{y}\,\exp\{-\alpha y\} =$$
$$\lim_{\alpha\to 0}\frac{b^2}{\sqrt{2\pi}}\,\frac{1}{\alpha^2+(bp)^2} = \frac{b^2}{\sqrt{2\pi}\,(bp)^2}. \quad (2.30)$$

Thus, to evaluate the Coulomb energy, we have to compute

$$\langle\Psi_2|\hat{V}_{\mathrm{Coul}}|\Psi_1\rangle = \frac{e^2}{b\sqrt{2\pi}}\left(\frac{\pi\sqrt{\pi}}{b^3}\right)\frac{1}{\pi\sqrt{\pi}}\int\mathrm{d}^3\vec{d}\,\frac{1}{\pi\sqrt{\pi}}$$
$$\cdot\int\mathrm{d}^3\vec{D}\left\{\frac{1}{d^2}\delta_{\sigma_1\sigma_3}\delta_{\sigma_2\sigma_4} - \frac{1}{D^2}\delta_{\sigma_1\sigma_4}\delta_{\sigma_2\sigma_3}\right\}\cdot(\mathrm{ME}) \quad (2.31)$$

where either $|\vec{\Psi}_1\rangle = |\vec{\Psi}_2\rangle = |\,\rangle$ or $|\vec{\Psi}_1\rangle = b_{H'h'}|\,\rangle$ and $|\vec{\Psi}_2\rangle = b_{Hh}|\,\rangle$. The shorthand notation (ME) stands for the matrix elements (2.25) in the first, and (2.27) or the normalized version of (2.28) in the second case. Evaluating (2.31), we obtain for the expectation values of the Coulomb interaction in the oscillator ground states $|\,\rangle$

$$E_0^{\mathrm{Coul}}(A) \equiv \langle\,|\hat{V}_{\mathrm{Coul}}|\,\rangle \equiv \frac{\langle\,|\hat{V}_{\mathrm{Coul}}\hat{C}_A(0)|\,\rangle}{\langle\,|\hat{C}_A(0)|\,\rangle} =$$
$$\frac{e^2}{b}\sqrt{\frac{2}{\pi}}\cdot\left\{\begin{array}{ll} 1, & \text{for } {}^4\mathrm{He} \\ \frac{83}{4}, & \text{for } {}^{16}\mathrm{O} \\ \frac{7905}{64}, & \text{for } {}^{40}\mathrm{Ca}\end{array}\right\}. \quad (2.32)$$

Since the Coulomb interaction acts only between the protons, the same energies are obtained for the neutron-hole cases. For the (non-vanishing) proton hole-hole matrix elements we obtain in the unprojected case

$$\langle\,|b^{\dagger}_{Hh}\hat{V}_{\rm Coul}b_{H'h'}\,|\,\rangle \;=\; \Delta^{\rm prot}_{h'h}\left[\delta_{H'H}\,E^{\rm Coul}_0(A) \;-\; \frac{e^2}{b}\sqrt{\frac{2}{\pi}} \cdot \left\{\begin{array}{lll} 1, & \text{for } H=H'=0s & \text{in } {}^4\text{He}\\ \frac{11}{2}, & \text{for } H=H'=0s & \text{in } {}^{16}\text{O}\\ \frac{61}{12}, & \text{for } H=H'=0p & \text{in } {}^{16}\text{O}\\ \frac{111}{8}, & \text{for } H=H'=0s & \text{in } {}^{40}\text{Ca}\\ \frac{611}{48}, & \text{for } H=H'=0p & \text{in } {}^{40}\text{Ca}\\ \frac{5693}{480}, & \text{for } H=H'=0d & \text{in } {}^{40}\text{Ca}\\ \frac{2333}{192}, & \text{for } H=H'=1s & \text{in } {}^{40}\text{Ca}\\ \frac{67}{48}\sqrt{\frac{3}{2}}, & \text{for } H=0s,\ H'=1s & \text{in } {}^{40}\text{Ca} \end{array}\right\}\right]. \quad (2.33)$$

On the other hand, we obtain for the normalized COM-projected proton hole-hole matrix elements

$$\langle (Hh)^{-1},\,(0)|\hat{V}_{\rm Coul}|(H'h')^{-1},\,(0)\rangle \;=\; \Delta^{\rm prot}_{h'h}\left[\delta_{H'H}\,E^{\rm Coul}_0(A) \;-\; \frac{e^2}{b}\sqrt{\frac{2}{\pi}} \cdot \left\{\begin{array}{lll} 1, & \text{for } H=H'=0s & \text{in } {}^4\text{He}\\ \frac{269}{48}, & \text{for } H=H'=0s & \text{in } {}^{16}\text{O}\\ \frac{61}{12}, & \text{for } H=H'=0p & \text{in } {}^{16}\text{O}\\ \frac{251981}{18048}, & \text{for } H=H'=0s & \text{in } {}^{40}\text{Ca}\\ \frac{1231}{96}, & \text{for } H=H'=0p & \text{in } {}^{40}\text{Ca}\\ \frac{5693}{480}, & \text{for } H=H'=0d & \text{in } {}^{40}\text{Ca}\\ \frac{2333}{192}, & \text{for } H=H'=1s & \text{in } {}^{40}\text{Ca}\\ \frac{871}{160}\sqrt{\frac{5}{47}}, & \text{for } H=0s,\ H'=1s & \text{in } {}^{40}\text{Ca} \end{array}\right\}\right]. \quad (2.34)$$

The differences in the Coulomb energy due to the COM projection are rather small. Using again $b = 1.8$ MeV for ^{16}O, we obtain that for the $0s$-proton holes the projected Coulomb energy is only -13 keV smaller than the unprojected one, while in ^{40}Ca (using $b = 2.0$ fm) we obtain -5 keV for both the $0\tilde{s}$- and the $0p$-proton holes, respectively.

2.4 Gaussian interactions

We shall now evaluate the expressions out of subsect. 2.2 for Gaussian-shaped interactions. Let us start with the central part (2.20) which we shall write as a linear combination of N Gaussians:

$$\hat{V}_{\rm c} \;\equiv\; \frac{1}{2}\sum_{i\neq j=1}^{A}\sum_{ST}\sum_{\nu=1}^{N} u^{\nu}_{ST}\hat{P}_{ST}\,\exp\{-(r_{ij}/\lambda_\nu)^2\}\,, \quad (2.35)$$

where for simplicity we have assumed that the ranges λ_ν do not depend on the considered spin-isospin channel. The Fourier-transform of the radial dependence is now

$$F_\nu[\sqrt{2}\vec{p}\,] \;=\; (2\pi)^{-3/2}\int {\rm d}^3\vec{r}_{12} \cdot\exp\{-i\sqrt{2}\vec{p}\cdot\vec{r}_{12}\}\exp\{-(r_{12}/\lambda_\nu)^2\} = b^3\left(\frac{\lambda^2_\nu}{2b^2}\right)^{3/2}\exp\left\{-\left(\frac{\lambda^2_\nu}{2b^2}\right)(b\vec{p}\,)^2\right\}. \quad (2.36)$$

Inserting (2.36) in (2.20), using (2.25), evaluating the integrals over $\vec{d}$ and $\vec{D}$ and performing the spin-isospin sums, we obtain for the expectation value of (2.35) in the oscillator ground-state configurations

$$V^{\rm c}_0 \;\equiv\; \langle\,|\hat{V}_{\rm c}|\,\rangle \;\equiv\; \frac{\langle\,|\hat{V}_{\rm c}\hat{C}_A(0)|\,\rangle}{\langle\,|\hat{C}_A(0)|\,\rangle} \;=\; \sum_{\nu=1}^{N}(1-\alpha^2_\nu)^{3/2} \cdot\left\{\begin{array}{ll} [3u^\nu_{01}+3u^\nu_{10}]\,, & \text{for } {}^4\text{He}\\ {}[3u^\nu_{01}+3u^\nu_{10}]\,10\,[1-\frac{3}{5}\alpha^2_\nu+\frac{3}{8}\alpha^4_\nu] & \\ \quad+(1-\alpha^2_\nu)[9u^\nu_{11}+u^\nu_{00}]\,6\,, & \text{for } {}^{16}\text{O}\\ {}[3u^\nu_{01}+3u^\nu_{10}]\,55\,[1-\frac{15}{11}\alpha^2_\nu+\frac{15}{11}\alpha^4_\nu & \\ \quad-\frac{63}{88}\alpha^6_\nu+\frac{189}{704}\alpha^8_\nu]+(1-\alpha^2_\nu)[9u^\nu_{11}+u^\nu_{00}] & \\ \quad\times 45\,[1-\frac{2}{3}\alpha^2_\nu+\frac{7}{12}\alpha^4_\nu], & \text{for } {}^{40}\text{Ca} \end{array}\right\}, \quad (2.37)$$

where we have introduced the shorthand notation

$$\alpha^2_\nu \;\equiv\; \frac{1}{1\,+\,\lambda^2_\nu/(2b^2)}\,. \quad (2.38)$$

For the non-vanishing matrix elements between the unprojected hole states we get with the help of (2.27)

see eq. (2.39) *on the next page*

while for the non-diagonal $0s$-$1s$ matrix element in ^{40}Ca one obtains

$$V^{\rm c,\,nor}_{0s1s} \;\equiv\; \langle\,|b^{\dagger}_{0sh}\hat{V}_{\rm c}b_{1sh}\,|\,\rangle \;=\; -\sum_{\nu=1}^{N}(1-\alpha^2_\nu)^{3/2} \cdot\frac{5}{4}\sqrt{\frac{3}{2}}\alpha^2_\nu\left\{[3u^\nu_{01}+3u^\nu_{10}]\Big[1-\frac{3}{2}\alpha^2_\nu+\frac{7}{8}\alpha^4_\nu\Big] +(1-\alpha^2_\nu)\,[9u^\nu_{11}+u^\nu_{00}]\right\}. \quad (2.40)$$

Using (2.28), normalized with the help of (2.63) and (2.64) out of ref. [1] one gets on the other hand for the COM projected hole-hole matrix elements

see eq. (2.41) *on the next page*

while for the non-diagonal $0\tilde{s}$-$1s$ matrix element in ^{40}Ca one obtains

$$V^{\rm c,\,pro}_{0\tilde{s}1s} \;\equiv\; \langle(0\tilde{s}h)^{-1},\,(0)|\hat{V}_{\rm c}|(1sh)^{-1},\,(0)\rangle \;=\; -\sum_{\nu=1}^{N}(1-\alpha^2_\nu)^{3/2}\cdot\frac{39}{8}\sqrt{\frac{5}{47}}\alpha^2_\nu\left\{[3u^\nu_{01}+3u^\nu_{10}] \cdot\Big[1-\frac{3}{2}\alpha^2_\nu+\frac{7}{8}\alpha^4_\nu\Big] \;+\;(1-\alpha^2_\nu)\,[9u^\nu_{11}+u^\nu_{00}]\right\}. \quad (2.42)$$

$$V_H^{\mathrm{c,\,nor}} \equiv \langle |b^\dagger_{Hh}\hat{V}_{\mathrm{c}} b_{Hh}|\rangle = V_0^{\mathrm{c}} - \sum_{\nu=1}^{N}(1-\alpha_\nu^2)^{3/2}$$

$$\cdot \left\{ \begin{array}{ll} \frac{1}{2}[3u^\nu_{01}+3u^\nu_{10}]\,, & \text{for } H=0s \quad \text{in } {}^4\mathrm{He} \\ \frac{5}{4}[3u^\nu_{01}+3u^\nu_{10}] + (1-\alpha_\nu^2)[9u^\nu_{11}+u^\nu_{00}]\,\frac{3}{4}\,, & \text{for } H=0s \quad \text{in } {}^{16}\mathrm{O} \\ \frac{5}{4}[3u^\nu_{01}+3u^\nu_{10}]\,[1-\frac{4}{5}\alpha_\nu^2+\frac{1}{2}\alpha_\nu^4] + (1-\alpha_\nu^2)[9u^\nu_{11}+u^\nu_{00}]\,\frac{3}{4}\,, & \text{for } H=0p \quad \text{in } {}^{16}\mathrm{O} \\ \frac{11}{4}[3u^\nu_{01}+3u^\nu_{10}]\,[1-\frac{6}{11}\alpha_\nu^2+\frac{15}{44}\alpha_\nu^4] + (1-\alpha_\nu^2)[9u^\nu_{11}+u^\nu_{00}]\,\frac{9}{4}\,, & \text{for } H=0s \quad \text{in } {}^{40}\mathrm{Ca} \\ \frac{11}{4}[3u^\nu_{01}+3u^\nu_{10}]\,[1-\alpha_\nu^2+\frac{5}{8}\alpha_\nu^4] & \\ \quad +(1-\alpha_\nu^2)[9u^\nu_{11}+u^\nu_{00}]\,\frac{9}{4}\,[1-\frac{5}{9}\alpha_\nu^2+\frac{35}{72}\alpha_\nu^4]\,, & \text{for } H=0p \quad \text{in } {}^{40}\mathrm{Ca} \\ \frac{11}{4}[3u^\nu_{01}+3u^\nu_{10}]\,[1-\frac{37}{22}\alpha_\nu^2+\frac{25}{11}\alpha_\nu^4-\frac{175}{88}\alpha_\nu^6+\frac{315}{352}\alpha_\nu^8] & \\ \quad +(1-\alpha_\nu^2)[9u^\nu_{11}+u^\nu_{00}]\frac{9}{4}\,[1-\frac{5}{6}\alpha_\nu^2+\frac{35}{36}\alpha_\nu^4]\,, & \text{for } H=1s \quad \text{in } {}^{40}\mathrm{Ca} \\ \frac{11}{4}[3u^\nu_{01}+3u^\nu_{10}]\,[1-\frac{37}{22}\alpha_\nu^2+\frac{161}{88}\alpha_\nu^4-\frac{91}{88}\alpha_\nu^6+\frac{63}{176}\alpha_\nu^8] & \\ \quad +(1-\alpha_\nu^2)[9u^\nu_{11}+u^\nu_{00}]\frac{9}{4}\,[1-\frac{5}{6}\alpha_\nu^2+\frac{49}{72}\alpha_\nu^4]\,, & \text{for } H=0d \quad \text{in } {}^{40}\mathrm{Ca} \end{array} \right\} \qquad (2.39)$$

$$V_H^{\mathrm{c,\,pro}} \equiv \langle (Hh)^{-1},\,(0)|\hat{V}_{\mathrm{c}}|(Hh)^{-1},\,(0)\rangle = V_0^{\mathrm{c}} - \sum_{\nu=1}^{N}(1-\alpha_\nu^2)^{3/2}$$

$$\cdot \left\{ \begin{array}{ll} \frac{1}{2}[3u^\nu_{01}+3u^\nu_{10}]\,, & \text{for } H=0s \quad \text{in } {}^4\mathrm{He} \\ \frac{5}{4}[3u^\nu_{01}+3u^\nu_{10}][1+\frac{1}{5}\alpha_\nu^2-\frac{1}{8}\alpha_\nu^4] + (1-\alpha_\nu^2)[9u^\nu_{11}+u^\nu_{00}]\,\frac{3}{4}\,, & \text{for } H=0s \quad \text{in } {}^{16}\mathrm{O} \\ \frac{5}{4}[3u^\nu_{01}+3u^\nu_{10}]\,[1-\frac{4}{5}\alpha_\nu^2+\frac{1}{2}\alpha_\nu^4] + (1-\alpha_\nu^2)[9u^\nu_{11}+u^\nu_{00}]\,\frac{3}{4}\,, & \text{for } H=0p \quad \text{in } {}^{16}\mathrm{O} \\ \frac{11}{4}[3u^\nu_{01}+3u^\nu_{10}]\,[1-\frac{265}{517}\alpha_\nu^2+\frac{335}{1034}\alpha_\nu^4-\frac{21}{4136}\alpha_\nu^6 & \\ \quad +\frac{63}{33088}\alpha_\nu^8] + (1-\alpha_\nu^2)[9u^\nu_{11}+u^\nu_{00}]\,\frac{9}{4}\,[1+\frac{2}{47}\alpha_\nu^2-\frac{7}{188}\alpha_\nu^4]\,, & \text{for } H=0\tilde{s} \quad \text{in } {}^{40}\mathrm{Ca} \\ \frac{11}{4}[3u^\nu_{01}+3u^\nu_{10}]\,[1-\frac{71}{77}\alpha_\nu^2+\frac{295}{616}\alpha_\nu^4+\frac{3}{22}\alpha_\nu^6-\frac{9}{176}\alpha_\nu^8] & \\ \quad +(1-\alpha_\nu^2)[9u^\nu_{11}+u^\nu_{00}]\,\frac{9}{4}\,[1-\frac{11}{21}\alpha_\nu^2+\frac{11}{24}\alpha_\nu^4]\,, & \text{for } H=0p \quad \text{in } {}^{40}\mathrm{Ca} \\ \frac{11}{4}[3u^\nu_{01}+3u^\nu_{10}]\,[1-\frac{37}{22}\alpha_\nu^2+\frac{25}{11}\alpha_\nu^4-\frac{175}{88}\alpha_\nu^6+\frac{315}{352}\alpha_\nu^8] & \\ \quad +(1-\alpha_\nu^2)[9u^\nu_{11}+u^\nu_{00}]\,\frac{9}{4}\,[1-\frac{5}{6}\alpha_\nu^2+\frac{35}{36}\alpha_\nu^4]\,, & \text{for } H=1s \quad \text{in } {}^{40}\mathrm{Ca} \\ \frac{11}{4}[3u^\nu_{01}+3u^\nu_{10}]\,[1-\frac{37}{22}\alpha_\nu^2+\frac{161}{88}\alpha_\nu^4-\frac{91}{88}\alpha_\nu^6+\frac{63}{176}\alpha_\nu^8] & \\ \quad +(1-\alpha_\nu^2)[9u^\nu_{11}+u^\nu_{00}]\,\frac{9}{4}\,[1-\frac{5}{6}\alpha_\nu^2+\frac{49}{72}\alpha_\nu^4]\,, & \text{for } H=0d \quad \text{in } {}^{40}\mathrm{Ca} \end{array} \right\} \qquad (2.41)$$

Left to be considered in the present section is the spin-orbit interaction (2.22) for the case of a Gaussian-shaped radial dependence g_T. In analogy to (2.35) we use here

$$\hat{V}_{ls} \equiv \frac{1}{2}\sum_{i\neq j=1}^{A}\sum_{T}\sum_{\nu=1}^{N} v_T^\nu \hat{P}_T \exp\{-(r_{ij}/\lambda_\nu)^2\}\vec{l}_{ij}\cdot\vec{S}_{ij}\,, \qquad (2.43)$$

where again for simplicity we have assumed that the ranges λ_ν (which usually are different from those of the central interaction) do not depend on the considered spin-isospin channel. The Fourier transform (2.24) yields then

$$G^\nu_{-\mu}[\sqrt{2}\vec{p},\,\vec{K}-\vec{k}] = (2\pi)^{-3/2}\int d^3\vec{r}_{12}\exp\{-i\sqrt{2}\vec{p}\cdot\vec{r}_{12}\}$$
$$\cdot\frac{1}{\sqrt{2}}\left(\vec{r}_{12}\times(\vec{K}-\vec{k})\right)_{-\mu}\exp\{-(r_{12}/\lambda_\nu)^2\} =$$
$$-ib^3\left(\frac{\lambda_\nu^2}{2b^2}\right)^{5/2}\left[\vec{d}\times\vec{D}\right]_{-\mu}\exp\left\{-\left(\frac{\lambda_\nu^2}{2b^2}\right)(b\vec{p})^2\right\}, \qquad (2.44)$$

where again $b\vec{p} = \vec{d}$ for the direct and $b\vec{p} = \vec{D}$ for the exchange term. It is easily seen that the spin-orbit interaction yields no contribution for the oscillator ground states (2.25) nor for the s-holes in (2.27) or (2.28). It only acts between the $0p$-holes in ^{16}O and the $0p$-holes and $0d$-holes in ^{40}Ca. Evaluating these matrix elements and using a spin-orbit coupled basis yields for ^{16}O

$$\begin{aligned}\langle\,|b^\dagger_{0p_{jm}\,\tau}\hat{V}_{ls}b_{0p_{j'm'}\,\tau'}\,|\,\rangle_{^{16}\mathrm{O}} \;\equiv\; & \\ \langle(0p_{jm}\,\tau)^{-1},\,(0)|\hat{V}_{ls}|(0p_{j'm'}\,\tau')^{-1},\,(0)\rangle_{^{16}\mathrm{O}} = & \\ -\,\delta_{j'j}\delta_{m'm}\delta_{\tau'\tau}\sum_{\nu=1}^{N}[1-\alpha_\nu^2]^{5/2} & \\ \cdot\left\{j(j+1)-\frac{11}{4}\right\}\left\{\frac{9}{4}v_1^\nu\,+\,\frac{5}{4}v_0^\nu[1-\alpha_\nu^2]\right\}, & \quad (2.45)\end{aligned}$$

where $j\,=\,1/2$ or $j\,=\,3/2$ and we have again used the definition (2.38). For this "non-spurious" hole states the "normal" and the COM-projected result are identical.

In ^{40}Ca we get again identical results for the $0d$-holes. Here,

$$\begin{aligned}\langle\,|b^\dagger_{0d_{jm}\,\tau}\hat{V}_{ls}b_{0d_{j'm'}\,\tau'}\,|\,\rangle_{^{40}\mathrm{Ca}} \;\equiv\; & \\ \langle(0d_{jm}\,\tau)^{-1},\,(0)|\hat{V}_{ls}|(0d_{j'm'}\,\tau')^{-1},\,(0)\rangle_{^{40}\mathrm{Ca}} = & \\ -\,\delta_{j'j}\delta_{m'm}\delta_{\tau'\tau}\sum_{\nu=1}^{N}[1-\alpha_\nu^2]^{5/2} & \\ \cdot\left\{j(j+1)-\frac{27}{4}\right\}\left\{\frac{27}{4}v_1^\nu\left[1-\frac{3}{2}\alpha_\nu^2+\frac{7}{8}\alpha_\nu^4\right]\right. & \\ \left.+\frac{11}{4}v_0^\nu[1-\alpha_\nu^2]\left[1-\frac{21}{22}\alpha_\nu^2+\frac{63}{88}\alpha_\nu^4\right]\right\}, & \quad (2.46)\end{aligned}$$

where $j = 3/2$ or $j = 5/2$. For the $0p$-holes, however, we get different results. In the unprojected case

$$\begin{aligned}\langle\,|b^\dagger_{0p_{jm}\,\tau}\hat{V}_{ls}b_{0p_{j'm'}\,\tau'}\,|\,\rangle_{^{40}\mathrm{Ca}} = & \\ -\,\delta_{j'j}\delta_{m'm}\delta_{\tau'\tau}\sum_{\nu=1}^{N}[1-\alpha_\nu^2]^{5/2}\left\{j(j+1)-\frac{11}{4}\right\} & \\ \cdot\left\{\frac{15}{2}v_1^\nu\left[1-\frac{3}{2}\alpha_\nu^2+\frac{7}{8}\alpha_\nu^4\right]\,+\,\frac{5}{2}v_0^\nu[1-\alpha_\nu^2]\right\}, & \quad (2.47)\end{aligned}$$

while for the COM-projected matrix element

$$\begin{aligned}\langle(0p_{jm}\,\tau)^{-1},\,(0)|\hat{V}_{ls}|(0p_{j'm'}\,\tau')^{-1},\,(0)\rangle_{^{40}\mathrm{Ca}} = & \\ -\,\delta_{j'j}\delta_{m'm}\delta_{\tau'\tau}\sum_{\nu=1}^{N}[1-\alpha_\nu^2]^{5/2}\left\{j(j+1)-\frac{11}{4}\right\} & \\ \cdot\left\{\frac{207}{28}v_1^\nu\left[1-\frac{3}{2}\alpha_\nu^2+\frac{7}{8}\alpha_\nu^4\right]\right. & \\ \left.+\,\frac{67}{28}v_0^\nu[1-\alpha_\nu^2]\left[1-\frac{21}{134}\alpha_\nu^2+\frac{63}{536}\alpha_\nu^4\right]\right\}. & \quad (2.48)\end{aligned}$$

2.5 Results and discussion for density-independent interactions

We shall start the discussion for the Hamiltonian (1.1) with Gaussian-shaped effective interactions by summerizing the results out of subsects. 2.1 to 2.4.

The simplest case is the $(0s)^4$ oscillator ground state of ^{4}He. For this we obtained

$$\begin{aligned}E_0^{\mathrm{nor}}(^4\mathrm{He}) \,\equiv\, E_0^{\mathrm{pro}}(^4\mathrm{He}) \,\equiv\, \frac{9}{4}\hbar\omega\,+\,\frac{e^2}{b}\sqrt{\frac{2}{\pi}} & \\ +\sum_{\nu=1}^{N_c}[1-\alpha_\nu^2]^{3/2}\,[3u_{01}^\nu\,+\,3u_{10}^\nu]\,, & \quad (2.49)\end{aligned}$$

while for the $0s$-holes with respect to it we got

$$\begin{aligned}E^{\mathrm{nor}}_{0s_{1/2m}\,\tau}(^4\mathrm{He}) \,\equiv\, E^{\mathrm{pro}}_{0s_{1/2m}\,\tau}(^4\mathrm{He}) \,\equiv & \\ E_0^{\mathrm{nor}}(^4\mathrm{He})\,-\,\frac{3}{4}\hbar\omega\,-\,\delta_{\tau p}\frac{e^2}{b}\sqrt{\frac{2}{\pi}} & \\ -\sum_{\nu=1}^{N_c}[1-\alpha_\nu^2]^{3/2}\,\frac{1}{2}\,[3u_{01}^\nu\,+\,3u_{10}^\nu]\,, & \quad (2.50)\end{aligned}$$

where N_c gives the number of Gaussians used for the central part of the interaction and the parameters α_ν^2 are given in terms of the corresponding ranges by eq. (2.38). The superscript "pro" indicates the projection into the COM rest frame and "nor" refers to the normal procedure where only the $1/A$ effect of $\hat{H}_{\mathrm{com}}$ has been considered.

Next comes the $(0s)^4(0p)^{12}$ oscillator ground state of ^{16}O. Here we got

$$\begin{aligned}E_0^{\mathrm{nor}}(^{16}\mathrm{O}) \,\equiv\, E_0^{\mathrm{pro}}(^{16}\mathrm{O}) \,\equiv\, \frac{69}{4}\hbar\omega\,+\,\frac{83}{4}\frac{e^2}{b}\sqrt{\frac{2}{\pi}} & \\ +\sum_{\nu=1}^{N_c}[1-\alpha_\nu^2]^{3/2}\left\{10\,[3u_{01}^\nu+3u_{10}^\nu]\left[1-\frac{3}{5}\alpha_\nu^2+\frac{3}{8}\alpha_\nu^4\right]\right. & \\ \left.+\,[1-\alpha_\nu^2]\,6\,[9u_{11}^\nu+u_{00}^\nu]\right\}. & \quad (2.51)\end{aligned}$$

For the non-spurious $0p$-holes in this nucleus we have obtained

$$\begin{aligned}E^{\mathrm{nor}}_{0p_{jm}\,\tau}(^{16}\mathrm{O}) \,\equiv\, E^{\mathrm{pro}}_{0p_{jm}\,\tau}(^{16}\mathrm{O}) \,\equiv & \\ E_0^{\mathrm{nor}}(^{16}\mathrm{O})\,-\,\frac{5}{4}\hbar\omega\,-\,\delta_{\tau p}\frac{61}{12}\frac{e^2}{b}\sqrt{\frac{2}{\pi}} & \\ -\sum_{\nu=1}^{N_c}[1-\alpha_\nu^2]^{3/2}\left\{\frac{5}{4}\,[3u_{01}^\nu\,+\,3u_{10}^\nu]\right. & \\ \left.\cdot\left[1-\frac{4}{5}\alpha_\nu^2+\frac{1}{2}\alpha_\nu^4\right]\,+\,[1-\alpha_\nu^2]\,\frac{3}{4}\,[9u_{11}^\nu\,+\,u_{00}^\nu]\right\} & \\ -\sum_{\nu=1}^{N_{ls}}[1-\bar{\alpha}_\nu^2]^{5/2}\,\left\{j(j+1)-\frac{11}{4}\right\} & \\ \cdot\left\{\frac{9}{4}v_1^\nu\,+\,[1-\bar{\alpha}_\nu^2]\,\frac{5}{4}v_0^\nu\right\}, & \quad (2.52)\end{aligned}$$

where the use of N_{ls} and $\bar{\alpha}_\nu^2$ in the spin orbit term indicates that the number of Gaussians as well as the ranges are here in general different from those of the central term. For the $0s$-holes in ^{16}O, on the other hand, the "normal" and COM-projected results are not identical. Here we have obtained

$$E^{\mathrm{nor}}_{0s_{1/2m}\,\tau}(^{16}\mathrm{O}) \equiv E^{\mathrm{nor}}_0(^{16}\mathrm{O}) - \frac{3}{4}\left(\frac{17}{15}\right)\hbar\omega - \delta_{\tau p}\frac{264}{48}\frac{e^2}{b}\sqrt{\frac{2}{\pi}} - \sum_{\nu=1}^{N_c}[1-\alpha_\nu^2]^{3/2} \cdot \left\{\frac{5}{4}\left[3u_{01}^\nu + 3u_{10}^\nu\right] + [1-\alpha_\nu^2]\,\frac{3}{4}\left[9u_{11}^\nu + u_{00}^\nu\right]\right\}, \quad (2.53)$$

for the unprojected case, while with projection into the COM rest frame

$$E^{\mathrm{pro}}_{0s_{1/2m}\,\tau}(^{16}\mathrm{O}) \equiv E^{\mathrm{nor}}_0(^{16}\mathrm{O}) - \frac{3}{4}\hbar\omega - \delta_{\tau p}\frac{269}{48}\frac{e^2}{b}\sqrt{\frac{2}{\pi}} - \sum_{\nu=1}^{N_c}[1-\alpha_\nu^2]^{3/2}\left\{\frac{5}{4}\left[3u_{01}^\nu + 3u_{10}^\nu\right]\left[1+\frac{1}{5}\alpha_\nu^2 - \frac{1}{8}\alpha_\nu^4\right] + [1-\alpha_\nu^2]\,\frac{3}{4}\left[9u_{11}^\nu + u_{00}^\nu\right]\right\}. \quad (2.54)$$

Last we consider the $(0s)^4(0p)^{12}(1s0d)^{24}$ oscillator ground state of ^{40}Ca. Here the result was

$$E^{\mathrm{nor}}_0(^{40}\mathrm{Ca}) \equiv E^{\mathrm{pro}}_0(^{40}\mathrm{Ca}) \equiv \frac{237}{4}\hbar\omega + \frac{7905}{64}\frac{e^2}{b}\sqrt{\frac{2}{\pi}} + \sum_{\nu=1}^{N_c}[1-\alpha_\nu^2]^{3/2}\left\{55\left[3u_{01}^\nu + 3u_{10}^\nu\right] \cdot \left[1-\frac{15}{11}\alpha_\nu^2 + \frac{15}{11}\alpha_\nu^4 - \frac{63}{88}\alpha_\nu^6 + \frac{189}{704}\alpha_\nu^8\right] + [1-\alpha_\nu^2]\,45\left[9u_{11}^\nu + u_{00}^\nu\right]\left[1-\frac{2}{3}\alpha_\nu^2 + \frac{7}{12}\alpha_\nu^4\right]\right\}. \quad (2.55)$$

For the non-spurious $1s$-holes one gets

$$E^{\mathrm{nor}}_{1s_{1/2m}\,\tau}(^{40}\mathrm{Ca}) \equiv E^{\mathrm{pro}}_{1s_{1/2m}\,\tau}(^{40}\mathrm{Ca}) \equiv E^{\mathrm{nor}}_0(^{40}\mathrm{Ca}) - \frac{7}{4}\hbar\omega - \delta_{\tau p}\frac{2333}{192}\frac{e^2}{b}\sqrt{\frac{2}{\pi}} - \sum_{\nu=1}^{N_c}[1-\alpha_\nu^2]^{3/2}\left\{\frac{11}{4}\left[3u_{01}^\nu + 3u_{10}^\nu\right] \cdot \left[1-\frac{37}{22}\alpha_\nu^2 + \frac{25}{11}\alpha_\nu^4 - \frac{175}{88}\alpha_\nu^6 + \frac{315}{352}\alpha_\nu^8\right] + [1-\alpha_\nu^2]\,\frac{9}{4}\left[9u_{11}^\nu + u_{00}^\nu\right]\left[1-\frac{5}{6}\alpha_\nu^2 + \frac{35}{36}\alpha_\nu^4\right]\right\} \quad (2.56)$$

and for the non-spurious $0d$-holes

$$E^{\mathrm{nor}}_{0d_{jm}\,\tau}(^{40}\mathrm{Ca}) \equiv E^{\mathrm{pro}}_{0d_{jm}\,\tau}(^{40}\mathrm{Ca}) \equiv E^{\mathrm{nor}}_0(^{40}\mathrm{Ca}) - \frac{7}{4}\hbar\omega - \delta_{\tau p}\frac{5693}{480}\frac{e^2}{b}\sqrt{\frac{2}{\pi}} - \sum_{\nu=1}^{N_c}[1-\alpha_\nu^2]^{3/2}\left\{\frac{11}{4}\left[3u_{01}^\nu + 3u_{10}^\nu\right] \cdot \left[1-\frac{37}{22}\alpha_\nu^2 + \frac{161}{88}\alpha_\nu^4 - \frac{91}{88}\alpha_\nu^6 + \frac{63}{176}\alpha_\nu^8\right] + [1-\alpha_\nu^2]\,\frac{9}{4}\left[9u_{11}^\nu + u_{00}^\nu\right]\left[1-\frac{5}{6}\alpha_\nu^2 + \frac{49}{72}\alpha_\nu^4\right]\right\} - \sum_{\nu=1}^{N_{ls}}[1-\bar{\alpha}_\nu^2]^{5/2}\left\{j(j+1)-\frac{27}{4}\right\} \cdot \left\{\frac{27}{4}v_1^\nu\left[1-\frac{3}{2}\bar{\alpha}_\nu^2 + \frac{7}{8}\bar{\alpha}_\nu^4\right] + [1-\bar{\alpha}_\nu^2]\,\frac{11}{4}v_0^\nu\left[1-\frac{21}{22}\bar{\alpha}_\nu^2 + \frac{63}{88}\bar{\alpha}_\nu^4\right]\right\}. \quad (2.57)$$

For the $0p$-holes the results with and without the projection into the COM rest frame are different. Without projection we obtain

$$E^{\mathrm{nor}}_{0p_{jm}\,\tau}(^{40}\mathrm{Ca}) \equiv E^{\mathrm{nor}}_0(^{40}\mathrm{Ca}) - \frac{5}{4}\left(\frac{203}{195}\right)\hbar\omega - \delta_{\tau p}\frac{1222}{96}\frac{e^2}{b}\sqrt{\frac{2}{\pi}} - \sum_{\nu=1}^{N_c}[1-\alpha_\nu^2]^{3/2}\left\{\frac{11}{4}\left[3u_{01}^\nu + 3u_{10}^\nu\right]\left[1-\alpha_\nu^2 + \frac{5}{8}\alpha_\nu^4\right] + [1-\alpha_\nu^2]\,\frac{9}{4}\left[9u_{11}^\nu + u_{00}^\nu\right]\left[1-\frac{5}{9}\alpha_\nu^2 + \frac{35}{72}\alpha_\nu^4\right]\right\} - \sum_{\nu=1}^{N_{ls}}[1-\bar{\alpha}_\nu^2]^{5/2}\left\{j(j+1)-\frac{11}{4}\right\} \cdot \left\{\frac{15}{2}v_1^\nu\left[1-\frac{3}{2}\bar{\alpha}_\nu^2 + \frac{7}{8}\bar{\alpha}_\nu^4\right] + [1-\bar{\alpha}_\nu^2]\,\frac{5}{2}v_0^\nu\right\}, \quad (2.58)$$

while with projection we get

$$E^{\mathrm{pro}}_{0p_{jm}\,\tau}(^{40}\mathrm{Ca}) \equiv E^{\mathrm{nor}}_0(^{40}\mathrm{Ca}) - \frac{5}{4}\hbar\omega - \delta_{\tau p}\frac{1231}{96}\frac{e^2}{b}\sqrt{\frac{2}{\pi}} - \sum_{\nu=1}^{N_c}[1-\alpha_\nu^2]^{3/2}\left\{\frac{11}{4}\left[3u_{01}^\nu + 3u_{10}^\nu\right] \cdot \left[1-\frac{71}{77}\alpha_\nu^2 + \frac{295}{616}\alpha_\nu^4 + \frac{3}{22}\alpha_\nu^6 - \frac{9}{176}\alpha_\nu^8\right]\right.$$

$$+ [1-\alpha_\nu^2]\,\frac{9}{4}\,[9u_{11}^\nu + u_{00}^\nu]\left[1-\frac{11}{21}\alpha_\nu^2+\frac{11}{24}\alpha_\nu^4\right]\Bigg\}$$
$$-\sum_{\nu=1}^{N_{ls}}[1-\bar{\alpha}_\nu^2]^{5/2}\left\{j(j+1)-\frac{11}{4}\right\}$$
$$\cdot\left\{\frac{207}{28}v_1^\nu\left[1-\frac{3}{2}\bar{\alpha}_\nu^2+\frac{7}{8}\bar{\alpha}_\nu^4\right]\right.$$
$$\left.+[1-\bar{\alpha}_\nu^2]\,\frac{67}{28}v_0^\nu\left[1-\frac{21}{134}\bar{\alpha}_\nu^2+\frac{63}{536}\bar{\alpha}_\nu^4\right]\right\}. \quad (2.59)$$

Differences are obtained for the $0s$-holes, too. Here we have obtained

$$E_{0s_{jm}\,\tau}^{\rm nor}(^{40}{\rm Ca}) \equiv E_0^{\rm nor}(^{40}{\rm Ca})$$
$$-\frac{3}{4}\left(\frac{41}{39}\right)\hbar\omega - \delta_{\tau p}\frac{250416}{18048}\frac{e^2}{b}\sqrt{\frac{2}{\pi}} - \sum_{\nu=1}^{N_c}[1-\alpha_\nu^2]^{3/2}$$
$$\cdot\left\{\frac{11}{4}\,[3u_{01}^\nu + 3u_{10}^\nu]\left[1-\frac{6}{11}\alpha_\nu^2+\frac{15}{44}\alpha_\nu^4\right]\right.$$
$$\left.+[1-\alpha_\nu^2]\,\frac{9}{4}\,[9u_{11}^\nu + u_{00}^\nu]\right\} \quad (2.60)$$

in the unprojected case while the Galilei-invariant result was

$$E_{0\tilde{s}_{jm}\,\tau}^{\rm pro}(^{40}{\rm Ca}) \equiv E_0^{\rm nor}(^{40}{\rm Ca})$$
$$-\frac{3}{4}\hbar\omega - \delta_{\tau p}\frac{251981}{18048}\frac{e^2}{b}\sqrt{\frac{2}{\pi}} - \sum_{\nu=1}^{N_c}[1-\alpha_\nu^2]^{3/2}$$
$$\cdot\left\{\frac{11}{4}\,[3u_{01}^\nu + 3u_{10}^\nu]\left[1-\frac{265}{517}\alpha_\nu^2\right.\right.$$
$$\left.+\frac{335}{1034}\alpha_\nu^4-\frac{21}{4136}\alpha_\nu^6+\frac{63}{33088}\alpha_\nu^8\right] + [1-\alpha_\nu^2]$$
$$\left.\cdot\frac{9}{4}[9u_{11}^\nu + u_{00}^\nu]\left[1+\frac{2}{47}\alpha_\nu^2-\frac{7}{188}\alpha_\nu^4\right]\right\}. \quad (2.61)$$

Finally, we come to the non-diagonal matrix element in ^{40}Ca. Here, in the unprojected case

$$(H_{\rm int})_{0s_{1/2m}\,1s_{1/2m},\,\tau}^{\rm nor} = \sqrt{\frac{3}{2}}\frac{20}{39}\hbar\omega + \sqrt{\frac{3}{2}}\frac{67}{48}\frac{e^2}{b}\sqrt{\frac{2}{\pi}}$$
$$+\sum_{\nu=1}^{N_c}[1-\alpha_\nu^2]\,\frac{5}{4}\alpha_\nu^2\left\{[3u_{01}^\nu+3u_{10}^\nu]\right.$$
$$\left.\cdot\left[1-\frac{3}{2}\alpha_\nu^2+\frac{7}{8}\alpha_\nu^4\right] + [1-\alpha_\nu^2]\,[9u_{11}^\nu+u_{00}^\nu]\right\}, \quad (2.62)$$

while with projection into the COM rest frame

$$(H_{\rm int})_{0\tilde{s}_{1/2m}\,1s_{1/2m},\,\tau}^{\rm pro} = 2\sqrt{\frac{5}{47}}\hbar\omega + \sqrt{\frac{5}{47}}\frac{871}{160}\frac{e^2}{b}\sqrt{\frac{2}{\pi}}$$
$$+\sum_{\nu=1}^{N_c}[1-\alpha_\nu^2]\,\frac{39}{8}\sqrt{\frac{5}{47}}\alpha_\nu^2\left\{[3u_{01}^\nu+3u_{10}^\nu]\right.$$
$$\left.\cdot\left[1-\frac{3}{2}\alpha_\nu^2+\frac{7}{8}\alpha_\nu^4\right] + [1-\alpha_\nu^2]\,[9u_{11}^\nu + u_{00}^\nu]\right\}. \quad (2.63)$$

Let us define single-particle energies for the hole states via

$$\epsilon_{nljm\tau}^{\rm nor}(A) \equiv E_0^{\rm nor}(A) - E_{nljm\tau}^{\rm nor}(A)\,,$$
$$\epsilon_{nljm\tau}^{\rm pro}(A) \equiv E_0^{\rm pro}(A) - E_{nljm\tau}^{\rm pro}(A)\,. \quad (2.64)$$

Then, using the unprojected results from above, we obtain

$$\sum_{nljm\tau\le F}\frac{1}{2}\left(\epsilon_{nljm\tau}^{\rm nor}(A) + \frac{A-1}{A}\,t_{nljm\tau}^{\rm nor}(A)\right) = E_0^{\rm nor}(A), \quad (2.65)$$

where the $t_{nljm\tau}^{\rm nor}(A)$ are given by eq. (2.12) and the $nljm\tau$ run over all states occupied in $|\rangle$. This formula is nothing but the Hartree-Fock prescription to calculate the total energy of a single determinant and is known as "Kolthun's sum rule" [7]. Note, that the factor $(A-1)/A$ in front of the kinetic hole energies comes from the fact that we have subtracted the COM Hamiltonian in (1.1). It is quite satisfying to see, that in the COM-projected case this sum rule is also fulfilled

$$\sum_{nljm\tau\le F}\frac{1}{2}\,S_{nljm\tau}^{\rm pro}\left(\epsilon_{nljm\tau}^{\rm pro}(A) + \frac{A-1}{A}\,t_{nljm\tau}^{\rm pro}(A)\right) =$$
$$E_0^{\rm pro}(A) = E_0^{\rm nor}(A)\,, \quad (2.66)$$

provided we use the projected hole spectroscopic factors out of eq. (3.10) of ref. [1] and the $t_{nljm\tau}^{\rm pro}(A)$ out of eq. (2.7). This is a rather good check of the consistency of the COM-projected results.

We shall now evaluate the formulas (2.49) to (2.61) using the so-called Brink-Boeker interaction B1 [8]. In its original form B1 is a purely central interaction consisting out of a linear combination of two Gaussians, one with range $\lambda_1 = 0.7$ fm, the other with $\lambda_2 = 1.4$ fm. The corresponding strengths are $u_{10}^1 = u_{01}^1 = 389.5$ MeV, $u_{00}^1 = u_{11}^1 = 801.591$ MeV, and, $u_{10}^2 = u_{01}^2 = -140.6$ MeV, $u_{00}^2 = u_{11}^2 = -3.82432$ MeV, respectively. To this force we added a short-range ($\bar{\lambda}_1 = 0.5$ fm) single Gaussian with $v_0^1 = v_1^1 = -2988.3297$ MeV for the spin-orbit interaction. This part has the same volume integral as the zero-range spin-orbit interaction used in the Gogny-force D1S [9] which we shall consider in sect. 3.

We start by displaying the energies (2.49), (2.51) and (2.55) for the oscillator ground states of ^{4}He, ^{16}O and ^{40}Ca, respectively, as functions of the oscillator length b in fig. 1. To fit all three curves in one plot, not the total energies but the energies per nucleon have been plotted. Extracting the values of the oscillator length b at the minima and calculating the root mean-square radii

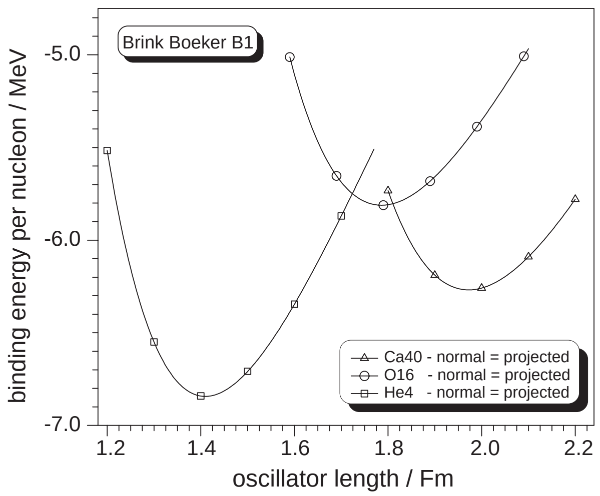

Fig. 1. The binding energies per nucleon obtained with the Brink-Boeker interaction B1 for the three even-even nuclei ^{4}He (eq. (2.49) divided by 4), ^{16}O (eq. (2.51) divided by 16) and ^{40}Ca (eq. (2.55) divided by 40) are displayed as functions of the oscillator length b. Here the interaction is Galilei-invariant and we consider "non-spurious" oscillator configurations. Since the COM Hamiltonian (1.3) has been subtracted, there is no difference between the "normal" and the "projected" description.

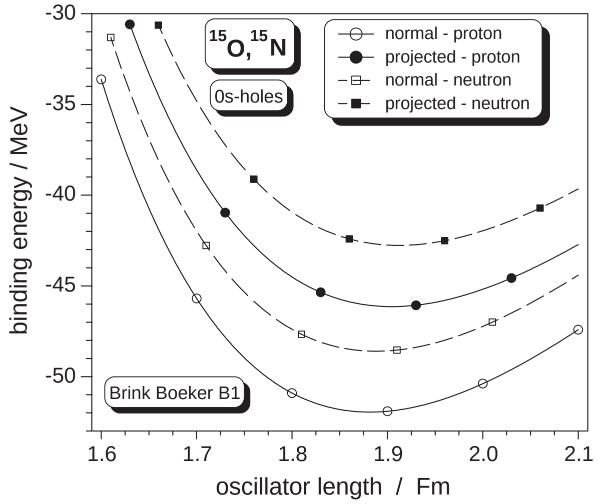

Fig. 2. The total binding energies for the 0s-holes in ^{16}O obtained with the B1 interaction in the normal (eq. (2.53), open symbols) and in the Galilei-invariant (eq. (2.54), full symbols) approach are displayed as functions of the oscillator length b. Full lines refer to proton holes; dashed lines to neutron holes.

one obtains reasonable agreement with the experimental data. On the other hand, it can be seen immediately that the binding energy per nucleon has not the experimentally observed A-dependence. From the Bethe-Weizsäcker mass formula we know that the binding energy per nucleon should increase up to a maximun at ^{56}Fe and hence we expect, to see more binding in ^{16}O than in ^{4}He and still even more in ^{40}Ca. This trend is not reproduced by the B1 interaction. Because we used non-spurious oscillator con-

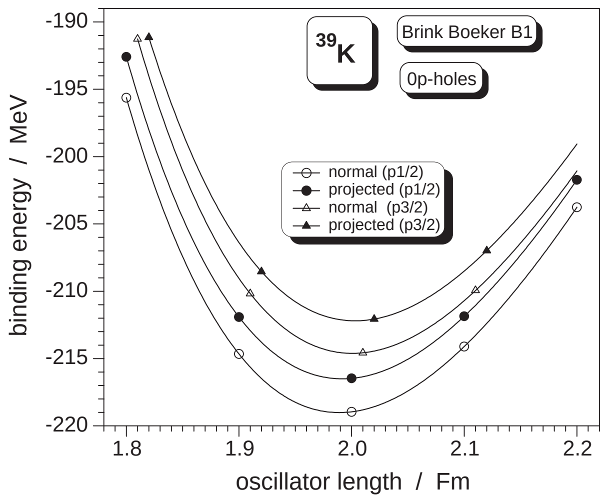

Fig. 3. The total binding energies for the 0p-proton holes in ^{40}Ca obtained with the B1 interaction complemented with the short-range two-body spin-orbit interaction of the Gogny force D1S in the normal (eq. (2.58), open symbols) and in the Galilei-invariant (eq. (2.59), full symbols) approach are displayed as functions of the oscillator length b. Circles refer to proton-$0p_{1/2}$-holes; triangles to proton-$0p_{3/2}$-holes.

figurations for the ground states of these three nuclei, the trivial $1/A$ effect which is accounted for by subtracting the COM Hamiltonian in (1.1) is the only effect of the restoration of Galilei invariance. No additional effects are induced here by the projection into the COM rest frame. The same holds for the, again non-spurious, one-hole states out of the last occupied major shells in these nuclei. Differences are, however, obtained for the one-hole states with excitation energies $\geq 1\hbar\omega$. Figure 2 shows the dependence of the total energies of the 0s-hole configurations in ^{16}O as a function of the oscillator length b. Compared are the unprojected solutions (2.53) for proton and neutron holes with the corresponding COM-projected results (2.54). As can be seen the COM-projected configurations are considerably less bound than the unprojected ones. At the corresponding minima one obtains around 6 MeV difference for both proton and neutron holes. Note, that this energy difference comes on top of the trivial $1/A$ effect which has already been considered in the "normal" approach.

We would also like to mention that for both the projected as well as unprojected results the oscillator lengths at the minima are about 0.1 fm larger than for the ^{16}O ground state. This so-called "rearrangement effect" yields energy gains for the hole states of the order of 1 to 2 MeV for the various 0s-hole configurations.

Similar effects are seen in ^{40}Ca. The energies of the 0p-proton hole configurations (2.58) and (2.59) are displayed in fig. 3. Here the rearrangement effect is much smaller. The oscillator lengths at the minima are here only about 0.02 fm larger than for the mother nucleus and the energy gains of the hole configurations only 0.2 to 0.5 MeV. Again the COM-projected configurations (2.59) are less bound than the unprojected ones (2.58). At the minima one obtains here around 2.5 MeV difference. The

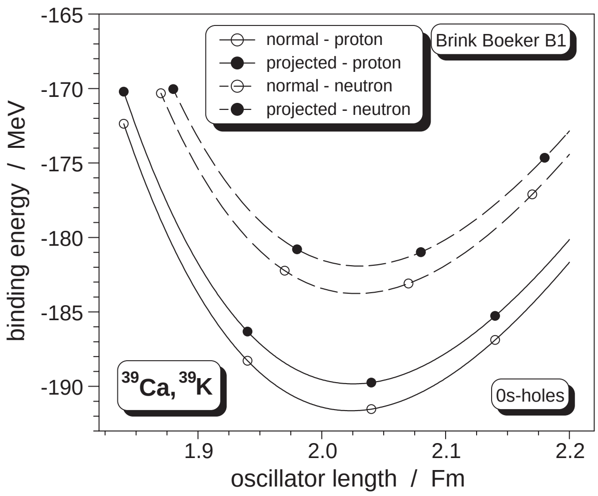

Fig. 4. The total binding energies for the 0s-holes in ^{40}Ca obtained with the B1 interaction in the normal (eq. (2.60), open symbols) and in the Galilei-invariant (eq. (2.61), full symbols) approach are displayed as functions of the oscillator length b. As in fig. 2 full lines refer to proton holes; dashed lines to neutron holes.

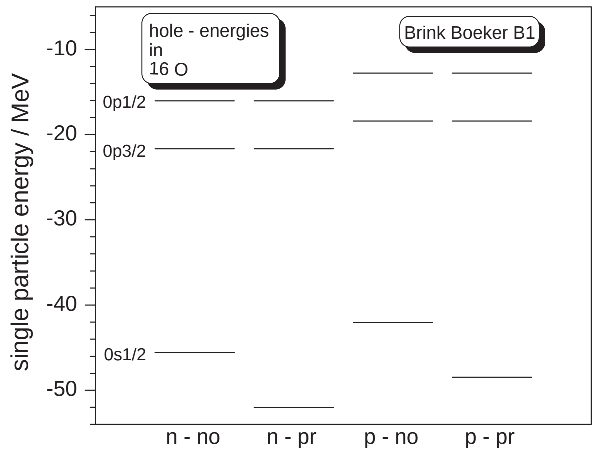

Fig. 5. The single-hole energies (2.64) of ^{16}O computed with the B1 interaction (plus spin-orbit term) are displayed for fixed oscillator length $b = 1.79$ fm. The neutron-hole states are displayed in the first two, the proton-hole states in the last two columns. "no" refers to the normal, "pr" to the Galilei-invariant description.

neutron holes which are not plotted in order to avoid an overload in the figure yield (except for an overall shift) almost the same result. Figure 4 displays the results for the 0s-hole configurations (2.60) and (2.61). Here again both the proton and the neutron results are shown. The rearrangement effect amounts here to about 0.05 fm in the oscillator lengths and about 0.3 to 0.5 MeV in the energies. Similar as for the 0p-states also here the COM-projected configurations are here by about 2.2 MeV less bound than the unprojected ones. The results for the single particle-energies (2.64) are summerized in fig. 5 for ^{16}O and fig. 6 for ^{40}Ca, respectively. For simplicity, here the rearrangement effects have been neglected and for all the states

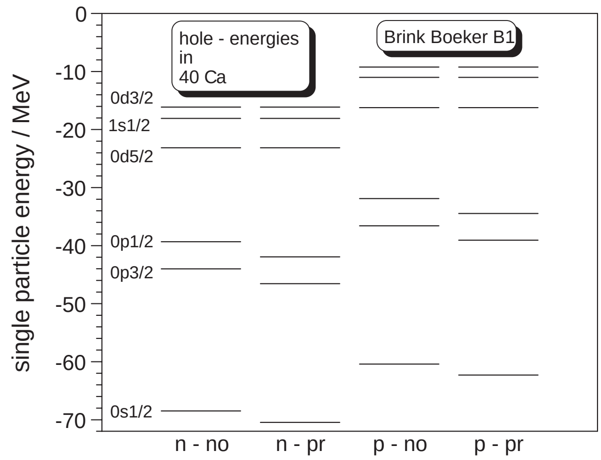

Fig. 6. Same as in fig. 5, but for the nucleus ^{40}Ca. Here a fixed oscillator length of $b = 1.97$ fm has been used.

fixed oscillator lengths of 1.79 fm in the first and 1.97 fm in the second case have been used. While for the non-spurious hole states the results obtained with and without the COM projection are identical, for the states with excitation energies $\geq 1\hbar\omega$ it costs considerably more energy to remove a "Galilei-invariant" nucleon than a normal one. This was to be expected because of the "depletion" of the occupation of these states seen in the spectroscopic factors (3.10) of ref. [1] which has to be compensated by a larger binding in order to fulfill the sum rule (2.66) which gives the same result as (2.65).

Unfortunately, already in the "normal" description the deep-lying hole states are too much bound as compared to the experimental situation. This, by the way, seems to be true for any Hamiltonian of the type (1.1), irrespective of the particular parametrisation used for the effective, density-independent interaction (1.6). This deficiency, together with the already-mentioned problems to reproduce the total binding energy and the root mean square radii of the nuclei simultaneously, led about 20 years ago to the proposal to use additional repulsive terms in the Hamiltonian. In order to "compress" the single-particle spectrum, the repulsion should be stronger for the deep-lying states (inside the nucleus) than for those near the Fermi level (at the surface of the nucleus) and thus it is intuitively clear that the repulsion should be proportional to the density (or a power of the density) of the nuclear medium. Forces of this type have been introduced by Skyrme [10], who used a zero-range version of eq. (1.6) plus a zero-range repulsion proportional to the density and were later on generalized by Gogny [6], who used a finite-range force of the type (2.35) for the central- and Skyrme's zero-range version of (2.43) for the spin-orbit term of (1.6) together with a zero-range repulsion proportional to the density to the power 1/3. Both forces have been applied rather successfully to many nuclei in the last two decades. So, *e.g.*, the binding energies as well as the form factors for elastic electron scattering could be reproduced by Hartree-Fock or Hartree-Fock-Bogoliubov calculations for many nuclei all over the mass table. Since furthermore microscopically derived effective interactions (*e.g.*, by G-matrix calculations for finite nuclei [5]) do also depend on the density of the surrounding nuclear medium (though much less strongly as the above-mentioned phenomenological interactions) it is obviously desirable to study the effects of the restoration of Galilei invariance also for such interactions. This will be done in the next section.

3 Energies with density-dependent interactions

In the following we shall investigate the second Hamiltonian (1.9) which includes a density-dependent interaction. For simplicity, we shall restrict ourselves to central interactions of the type (1.10). The extension to non-central interactions is straightforward.

3.1 Galilei-invariant form for density-dependent interactions

Normally, as already mentioned, instead of the Galilei-invariant form (1.10) the radial dependence

$$h_{ST}^{\rm nor} \equiv h_{ST}^{\rm nor}(\vec r_{ij}, \vec R_{ij}) \qquad (3.1)$$

is used. In the two-nucleon system a dependence on the COM coordinate of the two nucleons obviously is forbidden because of Galilei invariance. Inside the nucleus, however, it is possible, since Galilei invariance requires here only that the A-nucleon Hamiltonian should not depend on the total center-of-mass coordinate (1.2). We shall see that (3.1) does not fulfill this requirement while the radial dependence introduced in (1.10),

$$h_{ST}^{\rm inv} \equiv h_{ST}^{\rm inv}(\vec r_{ij}, \vec R_{ij} - \vec R_A), \qquad (3.2)$$

does.

Let us, nevertheless, start by writing the direct part of the two-body matrix elements of (3.1) in the plane-wave representation

$$\begin{aligned}\langle \vec k_1 \vec k_2 | h_{ST}^{\rm nor} | \vec k_3 \vec k_4\rangle &= \frac{1}{(2\pi)^6}\int d^3\vec r_i \int d^3\vec r_j \\ &\cdot \exp\{-i\vec k_1\cdot\vec r_i - i\vec k_2\cdot\vec r_j\}\, h_{ST}^{\rm nor}(\vec r_{ij},\vec R_{ij}) \\ &\cdot \exp\{i\vec k_3\cdot\vec r_i + i\vec k_4\cdot\vec r_j\} = \\ \frac{1}{(2\pi)^6}&\int d^3\vec r_{ij}\int d^3\vec R_{ij} \\ &\cdot \exp\{-i[(\vec k_1-\vec k_2)/2 - (\vec k_3-\vec k_4)/2]\cdot\vec r_{ij}\} \\ &\cdot \exp\{i[\vec k_1+\vec k_2-\vec k_3-\vec k_4]\cdot\vec R_{ij}\} h_{ST}^{\rm nor}(\vec r_{ij},\vec R_{ij}). \end{aligned} \qquad (3.3)$$

Introducing the relative and total momenta for the two-nucleon states (2.16) one obtains, for the direct term,

$$\begin{aligned}\langle \vec k_1 \vec k_2 | h_{ST}^{\rm nor} | \vec k_3 \vec k_4\rangle &= \frac{1}{(2\pi)^6}\int d^3\vec r_{ij}\exp\{-i(\vec q-\vec q\,')\cdot\vec r_{ij}\} \\ &\cdot \int d^3\vec R_{ij}\exp\{-i(\vec Q-\vec Q\,')\cdot\vec R_{ij}\} h_{ST}^{\rm nor}(\vec r_{ij},\vec R_{ij}) \equiv \\ &\frac{1}{(2\pi)^3} M_{ST}^{\rm nor}\left[\vec q-\vec q\,',\, \vec Q-\vec Q\,'\right], \end{aligned} \qquad (3.4)$$

while, for the exchange term,

$$\begin{aligned}\langle \vec k_1 \vec k_2 | h_{ST}^{\rm nor} | \vec k_4 \vec k_3\rangle &= \frac{1}{(2\pi)^6}\int d^3\vec r_{ij}\exp\{-i(\vec q+\vec q\,')\cdot\vec r_{ij}\} \\ &\cdot \int d^3\vec R_{ij}\exp\{-i(\vec Q-\vec Q\,')\cdot\vec R_{ij}\} h_{ST}^{\rm nor}(\vec r_{ij},\vec R_{ij}) \equiv \\ &\frac{1}{(2\pi)^3} M_{ST}^{\rm nor}\left[\vec q+\vec q\,',\, \vec Q-\vec Q\,'\right]. \end{aligned} \qquad (3.5)$$

Up to some constant factor, the matrix elements are thus the double Fourier transforms of the radial dependence (3.1).

Using (2.18), (2.19) and in addition

$$\vec T\,' \equiv \frac{1}{\sqrt 2}\vec Q\,', \qquad (3.6)$$

one obtains

$$\begin{aligned}\hat V_{\rm nor}^{\rm DD} \equiv \frac14\sum_{1234}\frac{1}{\pi\sqrt\pi}\int d^3\vec k\,\frac{1}{\pi\sqrt\pi}\int d^3\vec K\int d^3\vec T\int d^3\vec T\,' \\ \cdot\sum_{ST}\Big(W_{1234}^{ST} M_{ST}^{\rm nor}[\sqrt2\vec k,\,\sqrt2(\vec T-\vec T\,')] \\ - W_{1243}^{ST} M_{ST}^{\rm nor}[\sqrt2\vec K,\,\sqrt2(\vec T-\vec T\,')]\Big) \\ \cdot c^\dagger_{(\vec T+\vec K+\vec k)/\sqrt2\,1} c^\dagger_{(\vec T-\vec K-\vec k)/\sqrt2\,2} \\ \cdot c_{(\vec T\,'-\vec K+\vec k)/\sqrt2\,4} c_{(\vec T\,'+\vec K-\vec k)/\sqrt2\,3}, \end{aligned} \qquad (3.7)$$

where we have introduced analogously to (2.21)

$$W_{ijrs}^{ST} \equiv \langle ij | w_{ST}\hat P_{ST} | rs\rangle \qquad (3.8)$$

for the spin-isospin part of the interaction. Defining now

$$\vec t \equiv \frac12(\vec T - \vec T\,') \qquad \text{and} \qquad \vec s \equiv \frac12(\vec T + \vec T\,'), \qquad (3.9)$$

we obtain from (3.7)

$$\begin{aligned}\hat V_{\rm nor}^{\rm DD} \equiv \frac14\sum_{1234}\frac{1}{\pi\sqrt\pi}\int d^3\vec k\,\frac{1}{\pi\sqrt\pi}\int d^3\vec K\,(2^3)\int d^3\vec t \\ \cdot\sum_{ST}\Big(W_{1234}^{ST} M_{ST}^{\rm nor}[\sqrt2\vec k,\,2\sqrt2\vec t\,] \\ - W_{1243}^{ST} M_{ST}^{\rm nor}[\sqrt2\vec K,\,2\sqrt2\vec t\,]\Big) \\ \cdot\int d^3\vec s\, c^\dagger_{(\vec s+\vec t+\vec K+\vec k)/\sqrt2\,1} c^\dagger_{(\vec s+\vec t-\vec K-\vec k)/\sqrt2\,2} \\ \cdot c_{(\vec s-\vec t-\vec K+\vec k)/\sqrt2\,4} c_{(\vec s-\vec t+\vec K-\vec k)/\sqrt2\,3}. \end{aligned} \qquad (3.10)$$

This interaction is obviously not Galilei invariant. As can be easily seen, via the creation and annihilation operators the total momentum of the system is changed by $2\sqrt2\vec t$.

In order to obtain a Galilei-invariant formulation we have to take instead of (3.1) the radial dependence (3.2), which only depends on relative coordinates. We then introduce the double Fourier transform $M_{ST}^{\rm inv}[\vec\alpha,\,\vec\beta\,]$ of (3.2) via

$$h_{ST}^{\rm inv}(\vec r_{ij},\,\vec R_{ij}-\vec R_A)\;=\;\frac{1}{(2\pi)^3}\int{\rm d}^3\vec\alpha\int{\rm d}^3\vec\beta \cdot\exp\{i\vec\alpha\cdot\vec r_{ij}\}\exp\{i\vec\beta\cdot(\vec R_{ij}-\vec R_A)\}\,M_{ST}^{\rm inv}[\vec\alpha,\,\vec\beta\,]= \frac{1}{(2\pi)^3}\int{\rm d}^3\vec\alpha\int{\rm d}^3\vec\beta\,\exp\{i\vec\alpha\cdot\vec r_{ij}\}\exp\left\{i\frac{A-2}{A}\vec\beta\cdot\vec R_{ij}\right\} \cdot\exp\left\{-i\frac{A-2}{A}\vec\beta\cdot\vec R_{A-2}\right\}M_{ST}^{\rm inv}[\vec\alpha,\,\vec\beta\,]\,, \tag{3.11}$$

where

$$\vec R_{A-2}\;\equiv\;\frac{1}{A-2}\sum_{k\neq(i,j)=1}^{A}\vec r_k \tag{3.12}$$

is the center-of-mass coordinate of the remaining $A-2$ nucleons. Inserting (3.11) into (3.3), one gets

$$\langle\vec k_1\vec k_2|h_{ST}^{\rm inv}|\vec k_3\vec k_4\rangle\;=\;\frac{1}{(2\pi)^3}\int{\rm d}^3\vec\alpha\int{\rm d}^3\vec\beta\,\delta^{(3)} \cdot(\vec\alpha-(\vec q-\vec q\,'\,))\delta^{(3)}\left(\frac{A-2}{A}\vec\beta-(\vec Q-\vec Q\,'\,)\right) \cdot\exp\left\{-i\frac{A-2}{A}\vec\beta\cdot\vec R_{A-2}\right\}M_{ST}^{\rm inv}[\vec\alpha,\,\vec\beta\,]= \frac{1}{(2\pi)^3}\left(\frac{A}{A-2}\right)^3M_{ST}^{\rm inv}\left[\vec q-\vec q\,',\,\frac{A}{A-2}(\vec Q-\vec Q\,'\,)\right] \cdot\exp\left\{-i(\vec Q-\vec Q\,'\,)\cdot\vec R_{A-2}\right\}\,, \tag{3.13}$$

while for the exchange term (3.4), one obtains

$$\langle\vec k_1\vec k_2|h_{ST}^{\rm inv}|\vec k_4\vec k_3\rangle\;=\;\frac{1}{(2\pi)^3}\int{\rm d}^3\vec\alpha\int{\rm d}^3\vec\beta\,\delta^{(3)} \cdot(\vec\alpha-(\vec q+\vec q\,'\,))\delta^{(3)}\left(\frac{A-2}{A}\vec\beta-(\vec Q-\vec Q\,'\,)\right) \cdot\exp\left\{-i\frac{A-2}{A}\vec\beta\cdot\vec R_{A-2}\right\}M_{ST}^{\rm inv}[\vec\alpha,\,\vec\beta\,]= \frac{1}{(2\pi)^3}\left(\frac{A}{A-2}\right)^3M_{ST}^{\rm inv}\left[\vec q+\vec q\,',\,\frac{A}{A-2}(\vec Q-\vec Q\,'\,)\right] \cdot\exp\left\{-i(\vec Q-\vec Q\,'\,)\cdot\vec R_{A-2}\right\}\,. \tag{3.14}$$

Using again (2.18), we obtain for the Galilei-invariant form of the interaction

$$\hat V_{\rm inv}^{\rm DD}\;\equiv\;\frac14\sum_{1234}\int{\rm d}^3\vec k\int{\rm d}^3\vec K\int{\rm d}^3\vec Q\int{\rm d}^3\vec Q\,'\,\frac{1}{(2\pi)^3}\left(\frac{A}{A-2}\right)^3 \cdot\sum_{ST}\left\{W_{1234}^{ST}\,M_{ST}^{\rm inv}\left[\sqrt2\vec k,\,\frac{A}{A-2}(\vec Q-\vec Q\,'\,)\right]-W_{1243}^{ST}M_{ST}^{\rm inv}\left[\sqrt2\vec K,\,\frac{A}{A-2}(\vec Q-\vec Q\,'\,)\right]\right\} \cdot c^\dagger_{\vec Q/2+(\vec K+\vec k)/\sqrt2\;1}\,c^\dagger_{\vec Q/2-(\vec K+\vec k)/\sqrt2\;2} \cdot\exp\{-i(\vec Q-\vec Q\,'\,)\cdot\vec R_{A-2}\} \cdot c_{\vec Q\,'/2-(\vec K-\vec k)/\sqrt2\;4}\,c_{\vec Q\,'/2+(\vec K-\vec k)/\sqrt2\;3}\,. \tag{3.15}$$

We now introduce new variables

$$\vec z\;\equiv\;\frac12(\vec Q+\vec Q\,'\,)\qquad\text{and}\qquad\vec y\;\equiv\;\vec Q-\vec Q\,' \tag{3.16}$$

and furthermore commute the recoil operator with the two annihilators. Then (3.15) becomes

$$\hat V_{\rm inv}^{\rm DD}\;\equiv\;\frac14\sum_{1234}\int{\rm d}^3\vec k\int{\rm d}^3\vec K\int{\rm d}^3\vec z\int{\rm d}^3\vec y\,\frac{1}{(2\pi)^3}\left(\frac{A}{A-2}\right)^3 \cdot\sum_{ST}\left\{W_{1234}^{ST}\,M_{ST}^{\rm inv}\left[\sqrt2\vec k,\,\frac{A}{A-2}\vec y\right]-W_{1243}^{ST}M_{ST}^{\rm inv}\left[\sqrt2\vec K,\,\frac{A}{A-2}\vec y\right]\right\} \cdot c^\dagger_{\vec z/2+\vec y/4+(\vec K+\vec k)/\sqrt2\;1}\,c^\dagger_{\vec z/2+\vec y/4-(\vec K+\vec k)/\sqrt2\;2} \cdot c_{\vec z/2-(A+2)\vec y/(4A-8)-(\vec K-\vec k)/\sqrt2\;4} \cdot c_{\vec z/2-(A+2)\vec y/(4A-8)+(\vec K-\vec k)/\sqrt2\;3} \cdot\exp\left\{-i\frac{A}{A-2}\vec y\cdot\vec R_A\right\}\,. \tag{3.17}$$

Using now (for fixed $\vec y$)

$$\vec s\;\equiv\;\frac{1}{\sqrt2}\left(\vec z-\frac{1}{A-2}\vec y\right) \tag{3.18}$$

and afterwards

$$\vec t\;=\;\frac{1}{2\sqrt2}\frac{A}{A-2}\vec y, \tag{3.19}$$

we obtain finally

$$\hat V_{\rm inv}^{\rm DD}\;\equiv\;\frac14\sum_{1234}\frac{1}{\pi\sqrt\pi}\int{\rm d}^3\vec k\,\frac{1}{\pi\sqrt\pi}\int{\rm d}^3\vec K\,(2^3)\int{\rm d}^3\vec t \cdot\sum_{ST}\left\{W_{1234}^{ST}\,M_{ST}^{\rm inv}\left[\sqrt2\vec k,\,2\sqrt2\vec t\,\right]-W_{1243}^{ST}M_{ST}^{\rm inv}\left[\sqrt2\vec K,\,2\sqrt2\vec t\,\right]\right\} \cdot\int{\rm d}^3\vec s\,\cdot\,c^\dagger_{(\vec s+\vec t+\vec K+\vec k)/\sqrt2\;1}\,c^\dagger_{(\vec s+\vec t-\vec K-\vec k)/\sqrt2\;2} \cdot c_{(\vec s-\vec t-\vec K+\vec k)/\sqrt2\;4}\,c_{(\vec s-\vec t+\vec K-\vec k)/\sqrt2\;3} \cdot\exp\left\{-i\,2\sqrt2\vec t\cdot\vec R_A\right\}\,. \tag{3.20}$$

It is obvious that this form of the interaction is Galilei invariant: it leaves the total momentum of the considered A nucleon system unchanged.

Note, that (3.20) differs from (3.21) only by the recoil operator $\exp\{-i\,2\sqrt2\vec t\cdot\vec R_A\}$ and by the fact that the double Fourier transforms $M_{ST}^{\rm inv}$ of (3.2) instead of $M_{ST}^{\rm nor}$ of (3.1) are used. Let us therefore introduce (3.21) with the $M_{ST}^{\rm nor}$'s replaced by the $M_{ST}^{\rm inv}$'s

$$\hat{V}_{\rm mod}^{\rm DD} \equiv \frac{1}{4}\sum_{1234}\frac{1}{\pi\sqrt{\pi}}\int {\rm d}^3\vec{k}\,\frac{1}{\pi\sqrt{\pi}}\int {\rm d}^3\vec{K}\;(2^3)\int {\rm d}^3\vec{t}$$
$$\cdot\sum_{ST}\Bigg(W_{1234}^{ST}\,M_{ST}^{\rm inv}[\sqrt{2}\vec{k},\,2\sqrt{2}\vec{t}\,]$$
$$-\,W_{1243}^{ST}\,M_{ST}^{\rm inv}[\sqrt{2}\vec{K},\,2\sqrt{2}\vec{t}\,]\Bigg)$$
$$\cdot\int {\rm d}^3\vec{s}\,c^\dagger_{(\vec{s}+\vec{t}+\vec{K}+\vec{k})/\sqrt{2}\,1}\,c^\dagger_{(\vec{s}+\vec{t}-\vec{K}-\vec{k})/\sqrt{2}\,2}$$
$$\cdot\,c_{(\vec{s}-\vec{t}-\vec{K}+\vec{k})/\sqrt{2}\,4}\,c_{(\vec{s}-\vec{t}+\vec{K}-\vec{k})/\sqrt{2}\,3}\,. \qquad (3.21)$$

Then, comparing with (3.20), we have

$$\hat{V}_{\rm mod}^{\rm DD} \;=\; \hat{V}_{\rm inv}^{\rm DD}\;\exp\{i\,2\sqrt{2}\vec{t}\cdot\vec{R}_A\}\,. \qquad (3.22)$$

Now, the so-called "non-spurious" oscillator states for the A-nucleon system $|\psi_{\rm nso}^A\rangle$ have the property

$$|\psi_{\rm nso}^A\rangle \;=\; |\psi_{\rm int}^A\rangle\,|(0s)_{\rm COM}^A\rangle \qquad (3.23)$$

i.e., the center of mass is in a $0s$-oscillator state.

$$\langle(0s)_{\rm COM}^A|\exp\{i\,2\sqrt{2}\vec{t}\cdot\vec{R}_A\}|(0s)_{\rm COM}^A\rangle \;=$$
$$\frac{1}{\pi\sqrt{\pi}}\int {\rm d}^3\vec{X}\exp\{-X^2+iB\,2\sqrt{2}\vec{t}\cdot\vec{X}\} =$$
$$\exp\{-2\sqrt{2}B^2\,t^2/4\}\,, \qquad (3.24)$$

with

$$B^2 \;=\; \frac{\hbar}{MA\omega} \;=\; \frac{1}{A}b^2\,. \qquad (3.25)$$

Consequently we obtain

$$\langle\psi_{\rm nso}^A|\hat{V}_{\rm inv}^{\rm DD}|\psi_{\rm nso}^A\rangle \;=$$
$$\langle\psi_{\rm nso}^A|\hat{V}_{\rm mod}^{\rm DD}\exp\{2\sqrt{2}\,b^2\,t^2/(4A)\}|\psi_{\rm nso}^A\rangle\,. \qquad (3.26)$$

Thus, for non-spurious oscillator states we obtain a kind of "Tassie-Barker" factor [4] like for the form factors of electron scattering. Note, however, that this factor appears in the integrand and that furthermore for the evaluation of (3.26) the invariant double Fourier transforms $M_{ST}^{\rm inv}$ and not the usual expressions $M_{ST}^{\rm nor}$ have to be used.

3.2 The Gogny interaction

In the following we shall restrict ourselves to the density-dependent part of the Gogny interaction (1.12). Here the $\vec{r}_{ij}$ dependence is a Dirac delta-function. Thus, for the double Fourier transforms (3.4) and (3.5) of (1.12) we obtain

$$m_A^{\rm nor}[2\sqrt{2}\vec{t}\,] \;\equiv\; M_{ST}^{\rm nor}\left[\sqrt{2}\vec{k},\,2\sqrt{2}\vec{t}\,\right] \;=$$
$$M_{ST}^{\rm nor}\left[\sqrt{2}\vec{K},\,2\sqrt{2}\vec{t}\,\right] =$$
$$\frac{1}{(2\pi)^3}\int {\rm d}^3\vec{r}\,\exp\{-i\,2\sqrt{2}\vec{t}\cdot\vec{r}\}\left(\rho_A^{{\rm nor}\,(0)}(r)\right)^\alpha\,, \qquad (3.27)$$

where the superscript (0) does indicate that only the monopole part should be taken in order to conserve rotational invariance. Obviously via the density, (3.27) is mass and state dependent. For the corresponding invariant expressions (3.13) and (3.14) one gets

$$m_A^{\rm inv}[2\sqrt{2}\vec{t}\,] \;\equiv\; M_{ST}^{\rm inv}\left[\sqrt{2}\vec{k},\,2\sqrt{2}\vec{t}\,\right] \;=$$
$$M_{ST}^{\rm inv}\left[\sqrt{2}\vec{K},\,2\sqrt{2}\vec{t}\,\right] =$$
$$\frac{1}{(2\pi)^3}\int {\rm d}^3\vec{r}\,\exp\{-i\,2\sqrt{2}\vec{t}\cdot\vec{r}\}\left(\rho_A^{{\rm inv}\,(0)}(r)\right)^\alpha\,. \qquad (3.28)$$

Thus, for the Gogny force, (3.21) is replaced by

$$\hat{V}_{\rm nor}^{\rm DD}(A) \;\equiv\; \frac{1}{4}\sum_{1234}\left[W_{1234}\;-\;W_{1243}\right]\,(2^3)$$
$$\cdot\int {\rm d}^3\vec{t}\,m_A^{\rm nor}[2\sqrt{2}\vec{t}\,]\frac{1}{\pi\sqrt{\pi}}\int {\rm d}^3\vec{k}\,\frac{1}{\pi\sqrt{\pi}}\int {\rm d}^3\vec{K}$$
$$\cdot\int {\rm d}^3\vec{s}\,c^\dagger_{(\vec{s}+\vec{t}+\vec{K}+\vec{k})/\sqrt{2}\,1}\,c^\dagger_{(\vec{s}+\vec{t}-\vec{K}-\vec{k})/\sqrt{2}\,2}$$
$$\cdot\,c_{(\vec{s}-\vec{t}-\vec{K}+\vec{k})/\sqrt{2}\,4}\,c_{(\vec{s}-\vec{t}+\vec{K}-\vec{k})/\sqrt{2}\,3}\,, \qquad (3.29)$$

while for the invariant expression (3.20), one obtains

$$\hat{V}_{\rm inv}^{\rm DD}(A) \;\equiv\; \frac{1}{4}\sum_{1234}\left[W_{1234}\;-\;W_{1243}\right]\,(2^3)$$
$$\cdot\int {\rm d}^3\vec{t}\,m_A^{\rm inv}[2\sqrt{2}\vec{t}\,]\frac{1}{\pi\sqrt{\pi}}\int {\rm d}^3\vec{k}\,\frac{1}{\pi\sqrt{\pi}}\int {\rm d}^3\vec{K}$$
$$\cdot\int {\rm d}^3\vec{s}\,c^\dagger_{(\vec{s}+\vec{t}+\vec{K}+\vec{k})/\sqrt{2}\,1}\,c^\dagger_{(\vec{s}+\vec{t}-\vec{K}-\vec{k})/\sqrt{2}\,2}$$
$$\cdot\,c_{(\vec{s}-\vec{t}-\vec{K}+\vec{k})/\sqrt{2}\,4}\,c_{(\vec{s}-\vec{t}+\vec{K}-\vec{k})/\sqrt{2}\,3}$$
$$\cdot\,\exp\{-i\,2\sqrt{2}\vec{t}\cdot\vec{R}_A\;\}\,. \qquad (3.30)$$

In both these expressions now

$$W_{ijrs} \;\equiv\; \delta_{\tau_i\tau_r}\delta_{\tau_j\tau_s}t_0\langle\sigma_i\sigma_j|(1+x_0\hat{P}_\sigma)|\sigma_r\sigma_s\rangle\,, \qquad (3.31)$$

where $\hat{P}_\sigma$ is the spin-exchange operator.

Let us now first evaluate the expectation values of (3.29) and (3.30) within the simple oscillator ground-state configurations $|\,\rangle$ for ^{4}He, ^{16}O and ^{40}Ca. In the unprojected case, we obtain here

$$E_{\rm DD}^{\rm nor}(A) \;\equiv\; \langle\,|\hat{V}_{\rm nor}^{\rm DD}(A)|\,\rangle \;=\; \frac{1}{2}\sum_{12}\left[W_{1212}\;-\;W_{1221}\right]$$
$$\cdot\,\frac{8}{b^6}\,\frac{1}{\pi\sqrt{\pi}}\int {\rm d}^3\vec{z}\,\exp\{-z^2\}\,m_A^{\rm nor}\left[2\sqrt{2}\vec{z}\right]$$
$$\cdot\,\frac{1}{\pi\sqrt{\pi}}\int {\rm d}^3\vec{y}\,\exp\{-y^2\}\,\frac{1}{\pi\sqrt{\pi}}\int {\rm d}^3\vec{d}\,\exp\{-d^2\}$$
$$\cdot\,\frac{1}{\pi\sqrt{\pi}}\int {\rm d}^3\vec{D}\,\exp\{-D^2\}$$
$$\cdot\,y\left\{(\vec{y}-\vec{z}-\vec{D}+\vec{d})/\sqrt{2},\,(\vec{y}+\vec{z}-\vec{D}-\vec{d})/\sqrt{2}\right\}$$
$$\cdot\,y\left\{(\vec{y}-\vec{z}+\vec{D}-\vec{d})/\sqrt{2},\,(\vec{y}+\vec{z}+\vec{D}+\vec{d})/\sqrt{2}\right\}\,. \qquad (3.32)$$

Using the particular spin-isospin dependence of the Gogny force (3.31) we have

$$\frac{1}{2}\sum_{12}\left[W_{1212} - W_{1221}\right] = 6t_0 \qquad (3.33)$$

and thus, performing the last three Gaussian integrals with the functions y out of (2.26) from ref. [1], we obtain

$$E^{\rm nor}_{\rm DD}(A) = 6t_0\,\frac{8}{b^6}\,\frac{1}{\pi\sqrt{\pi}}\int d^3\vec{z}\,\exp\{-z^2\}\,m^{\rm nor}_A\left[2\sqrt{2}\vec{z}\right] \cdot\left\{\begin{array}{ll} 1, & \text{for } ^4\text{He} \\ \frac{31}{4} - 7z^2 + z^4, & \text{for } ^{16}\text{O} \\ \frac{1945}{64} - \frac{415}{8}z^2 + \frac{209}{8}z^4 - \frac{9}{2}z^6 + \frac{1}{4}z^8, & \text{for } ^{40}\text{Ca}\end{array}\right\}, \qquad (3.34)$$

where we have introduced $\vec{d} \equiv b\vec{k}$, $\vec{D} \equiv b\vec{K}$, $\vec{y} \equiv b\vec{s}$ and $\vec{z} \equiv b\vec{t}$, respectively. Using the definition (3.27) of the Fourier transform and $\vec{x} \equiv \vec{r}/b$, one immediately obtains

$$E^{\rm nor}_{\rm DD}(A) = \frac{6t_0\,b^{-3}}{\pi\sqrt{\pi}}\frac{1}{\pi\sqrt{\pi}}\int d^3\vec{x}\,e^{-2x^2}\left(\rho^{\rm nor\,(0)}_A(bx)\right)^\alpha \cdot\left\{\begin{array}{ll} 1, & \text{for } ^4\text{He} \\ 1 + 4x^2 + 4x^4, & \text{for } ^{16}\text{O} \\ \frac{25}{4} + 10x^4 + 4x^8, & \text{for } ^{40}\text{Ca}\end{array}\right\}. \qquad (3.35)$$

Since the $|\rangle$ are "non-spurious" oscillator configurations, the Galilei-invariant "projected" energies can be easily calculated using (3.26) in (3.34) and furthermore (3.28) instead of (3.27). Then, we obtain

$$\begin{aligned} E^{\rm pro}_{\rm DD}(A) &\equiv \frac{\langle\,|\hat{V}^{\rm DD}_{\rm inv}(A)\hat{C}_A(0)|\,\rangle}{\langle\,|\hat{C}_A(0)|\,\rangle} = \\ &\langle\,|\hat{V}^{\rm DD}_{\rm mod}(A)\exp\{2\sqrt{2}b^2t^2/(4A)\}|\,\rangle = \\ &\frac{6t_0b^{-3}}{\pi\sqrt{\pi}}\left(\frac{A}{A-2}\right)^{3/2}\frac{1}{\pi\sqrt{\pi}}\int d^3\vec{x} \\ &\cdot\exp\Big\{-2\frac{A}{A-2}x^2\Big\}\left(\rho^{\rm inv\,(0)}_A(bx)\right)^\alpha \\ &\cdot\left\{\begin{array}{ll} 1, & \text{for } ^4\text{He} \\ \frac{127}{196} + \frac{1152}{343}x^2 + \frac{16384}{2401}x^4, & \text{for } ^{16}\text{O} \\ \frac{52345945}{8340544} - \frac{3641500}{2476099}x^2 + \frac{592720000}{47045881}x^4 & \\ \quad - \frac{2304000000}{893871739}x^6 + \frac{102400000000}{16983563041}x^8, & \text{for } ^{40}\text{Ca}\end{array}\right\}. \end{aligned} \qquad (3.36)$$

Obviously this expression differs from the unprojected result (3.35). Thus, for density-dependent interactions effects of the restoration of Galilei invariance can already be seen in the energies of the doubly closed-shell oscillator ground states. Using (3.33) and in addition that, because of (3.31),

$$\sum_2\left[W_{h'\,2h2} - W_{h'\,22h}\right] = 3t_0\,\Delta_{h'h}\,, \qquad (3.37)$$

we obtain for the energies of the unprojected one-hole states

$$\begin{aligned} &E^{\rm nor}_{\rm DD}(A-1,\,HhH'h') \equiv \\ &\quad\langle\,|b^\dagger_{Hh}\hat{V}^{\rm DD}_{\rm nor}(A-1,\,H(m))b_{H'h'}|\,\rangle = \\ &\quad\Delta_{h'h}\,\frac{8}{b^6}\,\frac{1}{\pi\sqrt{\pi}}\int d^3\vec{z}\,\exp\{-z^2\}\,m^{\rm nor}_{A-1,\,H(m)}\left[2\sqrt{2}\vec{z}\right] \\ &\quad\cdot\frac{1}{\pi\sqrt{\pi}}\int d^3\vec{y}\,\exp\{-y^2\}\,\frac{1}{\pi\sqrt{\pi}}\int d^3\vec{d}\,\exp\{-d^2\} \\ &\quad\cdot\frac{1}{\pi\sqrt{\pi}}\int d^3\vec{D}\,\exp\{-D^2\} \\ &\quad\cdot y\left\{(\vec{y}-\vec{z}-\vec{D}+\vec{d})/\sqrt{2},\,(\vec{y}+\vec{z}-\vec{D}-\vec{d})/\sqrt{2}\right\} \\ &\quad\cdot\Big\{6t_0\,\delta_{H'H}\,y\left\{(\vec{y}-\vec{z}+\vec{D}-\vec{d})/\sqrt{2},\,(\vec{y}+\vec{z}+\vec{D}+\vec{d})/\sqrt{2}\right\} \\ &\qquad - 3t_0\,(H'|(\vec{y}+\vec{z}+\vec{D}+\vec{d})/\sqrt{2}) \\ &\qquad\cdot((\vec{y}-\vec{z}+\vec{D}-\vec{d})/\sqrt{2}|H)\Big\} \end{aligned} \qquad (3.38)$$

It is an easy exercise to show that $H \neq H'$ can only occur for $H = 0s$ and $H' = 1s$ (or vice versa) in ^{40}Ca. In this special case the density has to be calculated using the mixed states

$$\begin{pmatrix} b_{0s(m)h}|\,\rangle \\ b_{1s(m)h}|\,\rangle \end{pmatrix} = \begin{pmatrix} +\cos\varphi & +\sin\varphi \\ -\sin\varphi & +\cos\varphi \end{pmatrix}\begin{pmatrix} b_{0sh}|\,\rangle \\ b_{1sh)}|\,\rangle \end{pmatrix} \qquad (3.39)$$

with the mixing angle to be determined self-consistently in an iterative procedure always diagonalizing the actual total Hamiltonian, then calculating the new density with the states (3.39), which gives an improved Hamiltonian, etc., until convergence is achieved. This is indicated by the index $H(m)$. Note, that the first term in (3.38) is *not* equal to (3.34) as it was the case for density-independent interactions. This is due to the fact that the density-dependent term is mass and state dependent and thus in (3.34) and (3.38) *different* densities have to be used. Evaluating (3.38) in the same way as above, one obtains for the non-vanishing matrix elements

$$\begin{aligned} E^{\rm nor}_{\rm DD}(A,\,(Hh)^{-1}) &= \frac{3t_0\,b^{-3}}{\pi\sqrt{\pi}}\frac{1}{\pi\sqrt{\pi}} \\ &\cdot\int d^3\vec{x}\,\exp\{-2x^2\}\left(\rho^{\rm nor\,(0)}_{A-1\,Hh(m)}(bx)\right)^\alpha \\ &\cdot\left\{\begin{array}{ll} 1, & \text{for } H = 0s \text{ in } ^4\text{He} \\ 1 + 6x^2 + 8x^4, & \text{for } H = 0s \text{ in } ^{16}\text{O} \\ 2 + \frac{22}{3}x^2 + \frac{20}{3}x^4, & \text{for } H = 0p \text{ in } ^{16}\text{O} \\ 10 + 18x^4 + 8x^8 & \text{for } H = 0s \text{ in } ^{40}\text{Ca} \\ \frac{25}{2} - \frac{5}{3}x^2 + 20x^4 - \frac{4}{3}x^6 + 8x^8, & \text{for } H = 0p \text{ in } ^{40}\text{Ca} \\ \frac{35}{4} + 5x^2 + \frac{46}{3}x^4 + 4x^6 + \frac{20}{3}x^8, & \text{for } H = 1s \text{ in } ^{40}\text{Ca} \\ \frac{25}{2} + \frac{58}{3}x^4 + \frac{112}{15}x^8, & \text{for } H = 0d \text{ in } ^{40}\text{Ca}\end{array}\right\}, \end{aligned} \qquad (3.40)$$

$$
\begin{aligned}
&E^{\rm pro}_{\rm DD}(A,\,(Hh)^{-1}) \,=\, \langle (Hh)^{-1},\,(0)|\hat{V}^{\rm DD}_{\rm inv}(A-1,H(m))|(Hh)^{-1},\,(0)\rangle \,=\\
&\left(\frac{A-1}{A-3}\right)^{3/2}\frac{3t_0\,b^{-3}}{\pi\sqrt{\pi}}\frac{1}{\pi\sqrt{\pi}}\int {\rm d}^3\vec{x}\,\exp\left\{-2\frac{A-1}{A-3}x^2\right\}\left(\rho^{\rm inv\,(0)}_{A-1\,Hh(m)}(bx)\right)^{\alpha}\\
&\cdot\left\{\begin{array}{ll}
1, & \text{for } H=0s \text{ in } {}^4\text{He}\\
\frac{1445}{2197}+\frac{145675}{28561}x^2+\frac{4424625}{371293}x^4+\frac{2025000}{4826809}x^6, & \text{for } H=0s \text{ in } {}^{16}\text{O}\\
\frac{220}{169}+\frac{39060}{16393}x^2+\frac{2646000}{1590121}x^4, & \text{for } H=0p \text{ in } {}^{16}\text{O}\\
\frac{1178029585243213}{115765575574080}-\frac{1005036913988783}{356943858020080}x^2+\frac{76013589735564897}{3301730686685740}x^4-\frac{155980355576743971}{30541008851843095}x^6 & \\
\quad+\frac{13368695566716334494}{1130017327518194515}x^8+\frac{552099003286525344}{8362128223634639411}x^{10}+\frac{173661996484087488}{1546993721372408291035}x^{12}, & \text{for } H=0\tilde{s} \text{ in } {}^{40}\text{Ca}\\
\frac{70029036007}{5547516560}-\frac{128233821469}{35920169726}x^2+\frac{81523463308326}{3322615699655}x^4-\frac{697881412389564}{122936780887235}x^6 & \\
\quad+\frac{52240001774508288}{4548660892827695}x^8+\frac{13358615114160576}{168300453034624715}x^{10}, & \text{for } H=0p \text{ in } {}^{40}\text{Ca}\\
\frac{16096180}{1874161}+\frac{229889010}{69343957}x^2+\frac{46424591694}{2565726409}x^4+\frac{98524825308}{94931877133}x^6+\frac{35680061736540}{3512479453921}x^8, & \text{for } H=1s \text{ in } {}^{40}\text{Ca}\\
\frac{23526706}{1874161}-\frac{203072766}{69343957}x^2+\frac{314314890318}{12828632045}x^4-\frac{2364595807392}{474659385665}x^6+\frac{199808345724624}{17562397269605}x^8, & \text{for } H=0d \text{ in } {}^{40}\text{Ca}
\end{array}\right\}
\end{aligned}
\tag{3.44}
$$

while for the non-diagonal matrix element we get

$$
\begin{aligned}
&E^{\rm nor}_{\rm DD}(A=40,\,(0sh)^{-1}-(1sh)^{-1}) \,=\\
&\cdot -\frac{3t_0\,b^{-3}}{\pi\sqrt{\pi}}\frac{1}{\pi\sqrt{\pi}}\int {\rm d}^3\vec{x}\,\exp\{-2x^2\}\\
&\cdot\left(\rho^{\rm nor\,(0)}_{A-1\,Hh(m)}(bx)\right)^{\alpha}\left\{\frac{5}{2}-\frac{5}{3}x^2+2x^4-\frac{4}{3}x^6\right\}.
\end{aligned}
\tag{3.41}
$$

We shall now evaluate the corresponding expressions with projection into the COM rest frame using the Galilei-invariant form of the Gogny interaction. For the not yet normalized matrix elements we get, after some suitable variable transformtions, instead of (3.38),

$$
\begin{aligned}
&\langle\,|b^{\dagger}_{Hh}\hat{V}^{\rm DD}_{\rm inv}(A-1,\,H(m))\hat{C}_{A-1}(0)b_{H'h'}\,|\,\rangle \,=\\
&\Delta_{h'h}\,\left(\frac{4}{A-3}\right)^{3/2} b^3\,\pi\sqrt{\pi}\,\frac{8}{b^6}\\
&\cdot\frac{1}{\pi\sqrt{\pi}}\int {\rm d}^3\vec{z}\,\exp\{-z^2\}\,m^{\rm inv}_{A-1,\,H(m)}\left[\frac{2\sqrt{2}}{b}\sqrt{\frac{A-1}{A-3}}\vec{z}\right]\\
&\cdot\frac{1}{\pi\sqrt{\pi}}\int {\rm d}^3\vec{y}\,\exp\{-y^2\}\,\frac{1}{\pi\sqrt{\pi}}\int {\rm d}^3\vec{u}\,\exp\{-u^2\}\\
&\cdot\frac{1}{\pi\sqrt{\pi}}\int {\rm d}^3\vec{v}\,\exp\{-v^2\}\,\frac{1}{\pi\sqrt{\pi}}\int {\rm d}^3\vec{w}\,\exp\{-w^2\}\\
&\cdot x(\vec{\beta}_4{}',\,\vec{\beta}_2)\Big\{6t_0\,z^{-1}_{H'H}(\vec{\beta},\,\vec{\beta}\,')x(\vec{\beta}_3{}',\,\vec{\beta}_1)\\
&\qquad -3t_0\,\tilde{r}_{H'}(\vec{\beta}_1,\,\vec{\beta}\,')r_H(\vec{\beta}_3{}',\,\vec{\beta}\,)\Big\},
\end{aligned}
\tag{3.42}
$$

where the $z^{-1}_{H'H}$ are given by eqs. (2.43) to (2.45) of [1] while $\tilde{r}_{H'}$, r_H and x can be evaluated according to eqs. (2.47) to (2.53) out of [1]. Furthermore, here

$$
\begin{aligned}
\vec{\beta} \;&\equiv\; \sqrt{\frac{2}{A-1}}\,\vec{w}\,+\,\frac{2i}{\sqrt{(A-1)(A-3)}}\,\vec{z}\,,\\
\vec{\beta}\,' \;&\equiv\; \sqrt{\frac{2}{A-1}}\,\vec{w}\,-\,\frac{2i}{\sqrt{(A-1)(A-3)}}\,\vec{z}\,,\\
\vec{\beta}_1 \;&\equiv\; -i\vec{v}\,+\,i\sqrt{\frac{A-1}{A-3}}\,\vec{z}\,+\,i\sqrt{2}\,\vec{u}\,,\\
\vec{\beta}_2 \;&\equiv\; -i\vec{v}\,+\,i\sqrt{\frac{A-1}{A-3}}\,\vec{z}\,-\,i\sqrt{2}\,\vec{u}\,,\\
\vec{\beta}_3{}' \;&\equiv\; -i\vec{v}\,-\,i\sqrt{\frac{A-1}{A-3}}\,\vec{z}\,+\,i\sqrt{2}\,\vec{y}\,,\\
\vec{\beta}_4{}' \;&\equiv\; -i\vec{v}\,-\,i\sqrt{\frac{A-1}{A-3}}\,\vec{z}\,-\,i\sqrt{2}\,\vec{y}\,.
\end{aligned}
\tag{3.43}
$$

Evaluating these matrix elements, transforming into the spherical basis and normalizing them according to eqs. (2.63) and (2.64) from ref. [1], we obtain, instead of (3.40),

see eq. (3.44) *above*

while for the non-diagonal matrix element, we obtain, instead of (3.41),

$$
\begin{aligned}
&E^{\rm pro}_{\rm DD}(A=40,\,(0\tilde{s}h)^{-1}-(1sh)^{-1}) \,=\\
&\langle (0\tilde{s}h)^{-1},\,(0)|\hat{V}^{\rm DD}_{\rm inv}(A-1,H(m))|(1sh)^{-1},\,(0)\rangle \,=\\
&\left(\frac{A-1}{A-3}\right)^{3/2}\frac{3t_0\,b^{-3}}{\pi\sqrt{\pi}}\frac{1}{\pi\sqrt{\pi}}\int {\rm d}^3\vec{x}\\
&\cdot\exp\left\{-2\frac{A-1}{A-3}x^2\right\}\left(\rho^{\rm inv\,(0)}_{A-1\,Hh(m)}(bx)\right)^{\alpha}
\end{aligned}
$$

$$\cdot \frac{-1}{\sqrt{235}}\left\{\frac{215321977397}{4438013248} - \frac{1998046654721}{41051622544}x^2 + \frac{19117079900451}{379727508532}x^4 - \frac{65657563124751}{3512479453921}x^6 - \frac{595582568986860}{129961739795077}x^8 + \frac{1113217926180048}{4808584372417849}x^{10}\right\}. \tag{3.45}$$

For the explicit evaluation of the above formulae we need the space representation of the mass densities obtained with and without the projection into the COM rest frame. These can be obtained rather easily by Fourier transforming the results for the charge form factors discussed in ref. [3] and using unity for both the charge form factors of the proton and the neutron. It should be stressed, however, that in the Gogny interaction normally the bare densities without the Tassie-Barker modification are used. Thus, for the "normal" densities the Tassie-Barker factor should not be included here.

Performing the Fourier transformation we obtain for the unprojected oscillator ground states $|\rangle$

$$\rho_A^{\rm nor}(r) = A\frac{b^{-3}}{\pi\sqrt{\pi}}\exp\{-x^2\} \cdot \left\{\begin{array}{ll} 1\,, & \text{for } ^4\text{He} \\ \frac14 + \frac12 x^2\,, & \text{for } ^{16}\text{O} \\ \frac14 + \frac15 x^4\,, & \text{for } ^{40}\text{Ca} \end{array}\right\}. \tag{3.46}$$

It is easily checked that

$$\int d^3\vec{r}\,\rho_A^{\rm nor}(r) = A. \tag{3.47}$$

Furthermore, we can calculate easily the moments of (3.46)

$$\langle r^{2n}\rangle_A^{\rm nor} \equiv \frac{\int d^3\vec{r}\rho_A^{\rm nor}(r)r^{2n}}{\int d^3\vec{r}\rho_A^{\rm nor}(r)}. \tag{3.48}$$

Here, we obtain

$$\langle r^{2n}\rangle_A^{\rm nor} = \left(\frac{b^2}{2}\right)^n (2n+1)!! \cdot \left\{\begin{array}{ll} 1\,, & \text{for } ^4\text{He} \\ \frac{n+2}{2}\,, & \text{for } ^{16}\text{O} \\ \frac{n^2+4n+5}{5}\,, & \text{for } ^{40}\text{Ca} \end{array}\right\}. \tag{3.49}$$

With projection into the COM rest frame we obtain, on the other hand,

$$\rho_A^{\rm inv}(r) = A\frac{b^{-3}}{\pi\sqrt{\pi}}\left(\frac{A}{A-1}\right)^{3/2}\exp\{-Ax^2/(A-1)\} \cdot \left\{\begin{array}{ll} 1\,, & \text{for } ^4\text{He} \\ \frac15 + \frac{128}{225}x^2\,, & \text{for } ^{16}\text{O} \\ \frac{381}{1521} - \frac{1600}{59319}x^2 + \frac{51200}{2313441}x^4\,, & \text{for } ^{40}\text{Ca} \end{array}\right\}. \tag{3.50}$$

Again it is checked easily that

$$\int d^3\vec{r}\,\rho_A^{\rm inv}(r) = A\,, \tag{3.51}$$

while for the moments now

$$\langle r^{2n}\rangle_A^{\rm inv} = \left(\frac{b^2}{2}\right)^n (2n+1)!!\left(\frac{A-1}{A}\right)^n \cdot \left\{\begin{array}{ll} 1\,, & \text{for } ^4\text{He} \\ \frac{n+2}{2}\frac{16n+30}{15n+30}\,, & \text{for } ^{16}\text{O} \\ \frac{n^2+4n+5}{5}\frac{1600(n^2+4n+5)-200(n+2)+4}{1521(n^2+4n+5)}\,, & \text{for } ^{40}\text{Ca} \end{array}\right\} \tag{3.52}$$

Note, that because the total angular momentum of the states $|\rangle$ is always zero, (3.46) and (3.50) are already the scalar (monopole) parts of the densities so that no further reduction is needed. It should, futhermore, be stressed that the projected densities decrease with increasing radius faster than the unprojected ones do. This is reflected, *e.g.*, in smaller root mean-square radii: the nucleus is more "localized" if the COM motion is projected out. Note, however, that in the case of the non-spurious oscillator ground-state configurations considered here, the Tassie-Barker factor would do. Including this factor in the "normal" description would just give the COM-projected results.

For the one-hole states $b_{Hh}|\rangle$, we obtain without COM projection,

$$\left(\rho_{A-1}^{{\rm nor}\,(0)}(r)\right)_{HH} = (A-1)\frac{b^{-3}}{\pi\sqrt{\pi}}e^{-x^2} \cdot \left\{\begin{array}{ll} 1\,, & \text{for } H=0s \text{ in } ^4\text{He} \\ \frac15 + \frac{8}{15}x^2\,, & \text{for } H=0s \text{ in } ^{16}\text{O} \\ \frac{4}{15} + \frac{22}{45}x^2\,, & \text{for } H=0p \text{ in } ^{16}\text{O} \\ \frac{3}{13} + \frac{8}{39}x^4\,, & \text{for } H=0s \text{ in } ^{40}\text{Ca} \\ \frac{10}{39} - \frac{2}{117}x^2 + \frac{8}{39}x^4\,, & \text{for } H=0p \text{ in } ^{40}\text{Ca} \\ \frac{10}{39} + \frac{116}{585}x^4\,, & \text{for } H=0d \text{ in } ^{40}\text{Ca} \\ \frac{17}{78} + \frac{2}{39}x^2 + \frac{22}{117}x^4\,, & \text{for } H=1s \text{ in } ^{40}\text{Ca} \end{array}\right\}, \tag{3.53}$$

while for the $0s$-$1s$ mixing term in ^{40}Ca, we obtain

$$\left(\rho_{39}^{{\rm nor}\,(0)}(r)\right)_{0s1s} = -\frac{b^{-3}}{\pi\sqrt{\pi}}e^{-x^2}\sqrt{\frac32}\left(1-\frac23 x^2\right). \tag{3.54}$$

Again it is checked easily that

$$\int d^3\vec{r}\left(\rho_{A-1}^{{\rm nor}\,(0)}(r)\right)_{HH} = A-1\,, \tag{3.55}$$

$$\langle r^{2n}\rangle^{\mathrm{inv}\,(0)}_{A-1,\,H} = \left(\frac{b^2}{2}\right)^n (2n+1)!! \left(\frac{A-2}{A-1}\right)^n \cdot \left\{ \begin{array}{ll} 1\,, & \text{for } H=0s \text{ in } {}^4\mathrm{He} \\ \frac{2n+14}{14}\,, & \text{for } H=0s \text{ in } {}^{16}\mathrm{O} \\ \frac{2n+21}{21}\,, & \text{for } H=0p \text{ in } {}^{16}\mathrm{O} \\ \frac{117n^4+207462n^3+159177055n^2+614310566n+735010440}{735010440}\,, & \text{for } H=0\tilde{s} \text{ in } {}^{40}\mathrm{Ca} \\ \frac{39n^3+22208n^2+84248n+102885}{102885}\,, & \text{for } H=0p \text{ in } {}^{40}\mathrm{Ca} \\ \frac{1131n^2+4379n+5415}{5415}\,, & \text{for } H=0d \text{ in } {}^{40}\mathrm{Ca} \\ \frac{429n^2+1775n+2166}{2166}\,, & \text{for } H=1s \text{ in } {}^{40}\mathrm{Ca} \end{array} \right\}, \tag{3.61}$$

while the volume integral of (3.54) is vanishing. For completeness we give the moments, too. Here

$$\langle r^{2n}\rangle^{\mathrm{nor}\,(0)}_{A-1,\,H} = \left(\frac{b^2}{2}\right)^n (2n+1)!! \cdot \left\{ \begin{array}{ll} 1\,, & \text{for } H=0s \text{ in } {}^4\mathrm{He} \\ \frac{8n+15}{15}\,, & \text{for } H=0s \text{ in } {}^{16}\mathrm{O} \\ \frac{22n+45}{45}\,, & \text{for } H=0p \text{ in } {}^{16}\mathrm{O} \\ \frac{8n^2+32n+39}{39}\,, & \text{for } H=0s \text{ in } {}^{40}\mathrm{Ca} \\ \frac{24n^2+94n+117}{117}\,, & \text{for } H=0p \text{ in } {}^{40}\mathrm{Ca} \\ \frac{116n^2+464n+585}{585}\,, & \text{for } H=0d \text{ in } {}^{40}\mathrm{Ca} \\ \frac{22n^2+94n+117}{117}\,, & \text{for } H=1s \text{ in } {}^{40}\mathrm{Ca} \end{array} \right\}, \tag{3.56}$$

while for the non-diagonal matrix element one obtains

$$\langle r^{2n}\rangle^{\mathrm{nor}\,(0)}_{39,\,0s1s} = \left(\frac{b^2}{2}\right)^n (2n+1)!!\sqrt{\frac{2}{3}}\,n\,. \tag{3.57}$$

With COM projection we obtain for the one-hole states

$$\left(\rho^{\mathrm{inv}\,(0)}_{A-1}(r)\right)_{HH} = (A-1)\frac{b^{-3}}{\pi\sqrt{\pi}}\left(\frac{A-1}{A-2}\right)^{3/2} \exp\left\{-\frac{A-1}{A-2}\,x^2\right\} \cdot \left\{ \begin{array}{ll} 1\,, & \text{for } H=0s \text{ in } {}^4\mathrm{He} \\ \frac{11}{14}+\frac{15}{98}x^2\,, & \text{for } H=0s \text{ in } {}^{16}\mathrm{O} \\ \frac{6}{7}+\frac{5}{49}x^2\,, & \text{for } H=0p \text{ in } {}^{16}\mathrm{O} \\ \frac{182392931}{784011136}-\frac{103273157}{3724052896}x^2 +\frac{159832352823}{707570050240}x^4 +\frac{2037192417}{6721915477280}x^6 +\frac{90224199}{510865576273280}x^8\,, & \text{for } H=0\tilde{s} \text{ in } {}^{40}\mathrm{Ca} \\ \frac{14053}{54872}-\frac{42705}{1042568}x^2 +\frac{22222317}{99043960}x^4+\frac{771147}{1881835240}x^6\,, & \text{for } H=0p \text{ in } {}^{40}\mathrm{Ca} \\ \frac{371}{1444}-\frac{377}{13718}x^2+\frac{573417}{2606420}x^4\,, & \text{for } H=0d \text{ in } {}^{40}\mathrm{Ca} \\ \frac{625}{2888}+\frac{767}{27436}x^2+\frac{217503}{1042568}x^4\,, & \text{for } H=1s \text{ in } {}^{40}\mathrm{Ca} \end{array} \right\} \tag{3.58}$$

and

$$\left(\rho^{\mathrm{inv}\,(0)}_{39}(r)\right)_{0s1s} = -\frac{b^{-3}}{\pi\sqrt{\pi}}\left(\frac{A-1}{A-2}\right)^{3/2} \cdot \exp\left\{-\frac{A-1}{A-2}\,x^2\right\}\sqrt{\frac{5}{47}}\left(\frac{806871}{219488}-\frac{8849685}{4170272}x^2 -\frac{80139969}{396175840}x^4+\frac{90224199}{7527340960}x^6\right) \tag{3.59}$$

for the non-diagonal mixing term in ^{40}Ca. Again (3.58) fulfills the sum rule

$$\int \mathrm{d}^3\vec{r}\,\left(\rho^{\mathrm{inv}\,(0)}_{A-1}(r)\right)_{HH} = A-1\,, \tag{3.60}$$

while the volume integral of (3.59) is vanishing.

For the moments of the Galilei-invariant one-hole densities one obtains

see eq. (3.61) *above*

while for the non-diagonal matrix element

$$\langle r^{2n}\rangle^{\mathrm{inv}\,(0)}_{39,\,0\tilde{s}1s} = \left(\frac{b^2}{2}\right)^n (2n+1)!!\left(\frac{A-2}{A-1}\right)^n \cdot\sqrt{235}\left(\frac{27848n+1149n^2-117n^3}{19342380}\right). \tag{3.62}$$

Again, the COM-projected densities fall off faster than the unprojected ones and smaller root mean-square radii are obtained. However, here the COM-projected results can be reproduced using the Tassie-Barker factor only for the non-spurious one-hole states out of the last occupied shell, while for the holes with excitation energies $\geq 1\hbar\omega$ this is not the case.

We can now insert these densities in the formulas above, perform the trivial angle integration and do the remaining one-dimensional integration for $\alpha = 1/3$ with the help of some computer algebra program numerically. Then we obtain for (3.34) the final results

$$E^{\mathrm{nor}}_{\mathrm{DD}}(A) = \frac{t_0}{b^4}\cdot\left\{ \begin{array}{ll} 0.2707528213441683, & \text{for } {}^4\mathrm{He} \\ 2.5145712697351553, & \text{for } {}^{16}\mathrm{O} \\ 11.32945642837287, & \text{for } {}^{40}\mathrm{Ca} \end{array} \right\}, \tag{3.63}$$

while with projection into the COM rest frame we obtain for (3.36)

$$E^{\rm pro}_{\rm DD}(A) = \frac{t_0}{b^4}\cdot\left\{\begin{array}{l} 0.3363770114399974, \text{ for } {}^4\text{He} \\ 2.5845635332647206, \text{ for } {}^{16}\text{O} \\ 11.41560488654768, \quad \text{for } {}^{40}\text{Ca} \end{array}\right\}. \quad (3.64)$$

For the corresponding unprojected one-hole states (3.40)

$$E^{\rm nor}_{\rm DD}(A,\,(Hh)^{-1}) = \frac{t_0}{b^4} \cdot\left\{\begin{array}{ll} 0.1229976318079724\,, & \text{for } H=0s \text{ in } {}^4\text{He} \\ 2.0313723641272342\,, & \text{for } H=0s \text{ in } {}^{16}\text{O} \\ 2.1954668935690650\,, & \text{for } H=0p \text{ in } {}^{16}\text{O} \\ 10.31910480436779\,, & \text{for } H=0s \text{ in } {}^{40}\text{Ca} \\ 10.57591424970498\,, & \text{for } H=0p \text{ in } {}^{40}\text{Ca} \\ 10.72996460769541\,, & \text{for } H=1s \text{ in } {}^{40}\text{Ca} \\ 10.79316541696027\,, & \text{for } H=0d \text{ in } {}^{40}\text{Ca} \end{array}\right\}, \quad (3.65)$$

while with projection into the COM rest frame, we obtain for (3.44)

$$E^{\rm pro}_{\rm DD}(A,\,(Hh)^{-1}) = \frac{t_0}{b^4} \cdot\left\{\begin{array}{ll} 0.1683524357510309\,, & \text{for } H=0s \text{ in } {}^4\text{He} \\ 2.2060808459379243\,, & \text{for } H=0s \text{ in } {}^{16}\text{O} \\ 2.4312045356577934\,, & \text{for } H=0p \text{ in } {}^{16}\text{O} \\ 10.39086084526980\,, & \text{for } H=0\tilde{s} \text{ in } {}^{40}\text{Ca} \\ 10.75526558039913\,, & \text{for } H=0p \text{ in } {}^{40}\text{Ca} \\ 10.81454144450567\,, & \text{for } H=1s \text{ in } {}^{40}\text{Ca} \\ 10.87733257118675\,, & \text{for } H=0d \text{ in } {}^{40}\text{Ca} \end{array}\right\}. \quad (3.66)$$

In (3.65) and (3.66), for simplicity, we have neglected the dynamical mixing between the $0s$ and $1s$ holes in ^{40}Ca. The differences between the projected and unprojected results seems small, however, one has to take into account that t_0 is of order 10^3 so that considerable differences will be seen.

At first sight the decrease of the repulsion with increasing hole energy seen in both the projected and unprojected results seems to contradict our aim to compress the hole energy spectra. However, (3.65) and (3.66) are absolute energies. For the hole energies it is the differences of (3.63) and (3.65) or (3.64) and (3.66) which matter: In these differences a clear decrease of the repulsion is seen when going from the deeply bound $0s$-hole to those in the neighbourhood of the Fermi sea as intended. It is furthermore seen that in all cases the (total) repulsion is larger in the COM-projected case as if calculated in the normal fashion. The dominant effect in this deviation comes from the use of the Galilei-invariant instead of the normal density, however, also the recoil term contributes in the same direction. Considering only the exponentials (and thus neclecting the differences in the polynomial parts one obtains a kind of "Gaussian-overlap approximation" (GOA), in which

$$E^{\rm pro}_{\rm DD}(A,\,{\rm GOA}) = \frac{(A-1)\sqrt{A}}{(A-8/7)\sqrt{A-8/7}}\,E^{\rm nor}_{\rm DD}(A,\,{\rm GOA})\,. \quad (3.67)$$

Equation (3.67) may be a possible starting point to renormalize the Gogny parameter t_0. We shall come back to this problem later.

3.3 Results and discussion for the Gogny force

In this section we shall discuss the results obtained with the Gogny interaction D1S [9]. This interaction has a central part of the form (2.35). As in the case of the Brink-Boeker interaction a linear combination of two Gaussians is used. The ranges are $\lambda_1 = 0.7$ fm and $\lambda_2 = 1.2$ fm, respectively, and the corresponding strength parameters $u^1_{10} = -836.23$ MeV, $u^1_{01} = 190.83$ MeV, $u^1_{00} = -6231.43$ MeV, $u^1_{11} = -4.37$ MeV, while $u^2_{10} = -120.966$ MeV, $u^2_{01} = -119.624$ MeV, $u^2_{00} = 653.868$ MeV, and $u^2_{11} = 1.278$ MeV. Furthermore, Gogny-D1S contains a zero-range spin-orbit interaction which we replaced by the finite-range form (2.43) with a single Gaussian with range $\bar{\lambda}_1 = 0.5$ fm and strengths $v^1_1 = v^1_0 = -2988.32970$ MeV. This interaction has exactly the same volume integral as the zero-range D1S spin-orbit term. Finally, the density-dependent term (1.12) is given by $\alpha = 1/3$, $x_0 = 1$ and $t_0 = 1390.6$ MeV.

Before starting to discuss the differences induced by the projection into the COM rest frame, let us have a look at the Gogny results obtained in the "normal" fashion. Figure 7 displays the results obtained with the Gogny-D1S interaction for the oscillator ground states of ^{4}He, ^{16}O and ^{40}Ca without (eqs. (2.49), (2.51), (2.55) plus (3.63)) and with the projection into the COM rest frame (eqs. (2.49), (2.51), (2.55) plus (3.64)). As in fig. 1 we have plotted the binding energies per nucleon as functions of the oscillator length b. We shall come back to the COM-projected results later. It is obvious that now the unprojected results reproduce both the absolute binding energies as well as their mass dependence rather well. This is no surprise, since the parameters of this phenomenological interaction were adjusted to do so. The same holds for the single-hole energies in ^{40}Ca which are compared to the B1 results in fig. 8. The spectra refer all to the unprojected ("normal") approach and have been obtained via (2.64) at the minimum b-values out of fig. 7. As expected, the additional repulsion of the deep-lying hole states leads indeed to the desired compression of the spectrum. Note that within the last occupied shell (the $1s0d$ one) almost no differences with respect to the B1 results are observed.

It is instructive to recall how these rather satisfying results are achieved via the density-dependent term. For this purpose we have plotted in fig. 9 the contributions of the different terms of the interaction to the binding energy per nucleon as functions of the mass number. Again the b-values at the minima of fig. 7 have been used. As can be easily seen, the dominant contribution comes here from

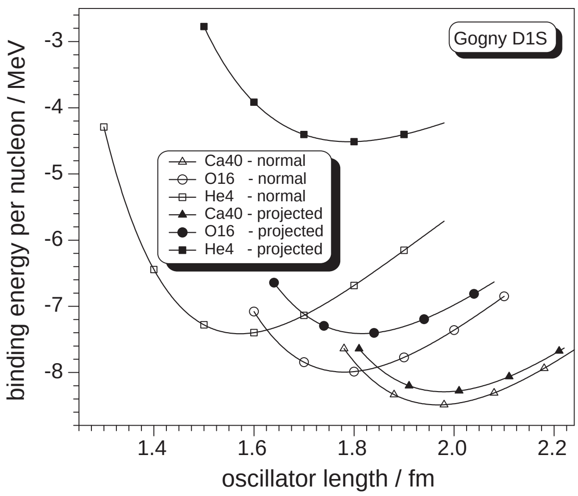

Fig. 7. The binding energies per nucleon obtained with the Gogny interaction D1S for the three even-even nuclei ^{4}He, ^{16}O and ^{40}Ca (eqs. (2.49), (2.51) and (2.55) plus (3.63) divided by A in the normal (open symbols) and eqs. (2.49), (2.51) and (2.55) plus (3.64) divided by A in the COM-projected description (full symbols)) are displayed as functions of the oscillator length b. Since the normal form of the density-dependent term of this interaction is *not* Galilei-invariant, we get here different results though we consider "non-spurious" oscillator configurations and the COM Hamiltonian (1.3) has been subtracted.

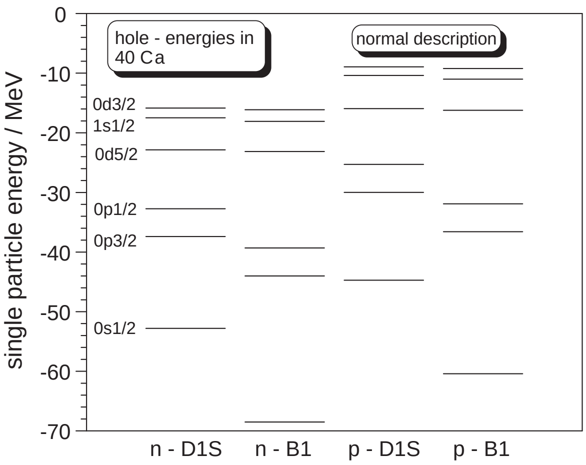

Fig. 8. The hole energies (2.64) in ^{40}Ca obtained in the normal approximation with the D1S interaction are compared to those computed with the Brink-Boeker force B1 out of fig. 6. For b again the fixed value of 1.97 fm has been used.

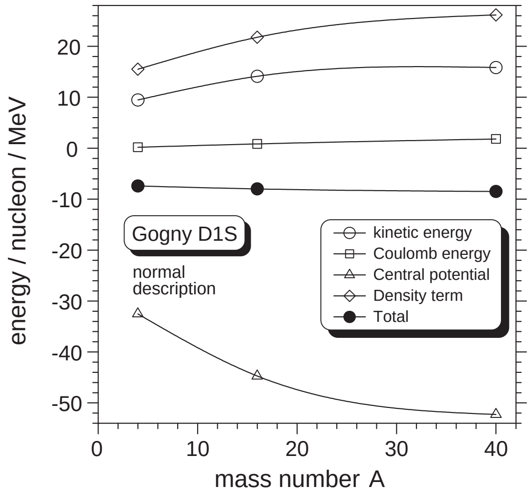

Fig. 9. The normal results for the binding energies per nucleon obtained with the Gogny interaction D1S out of fig. 7 are decomposed into their various contributions and plotted *versus* the mass number.

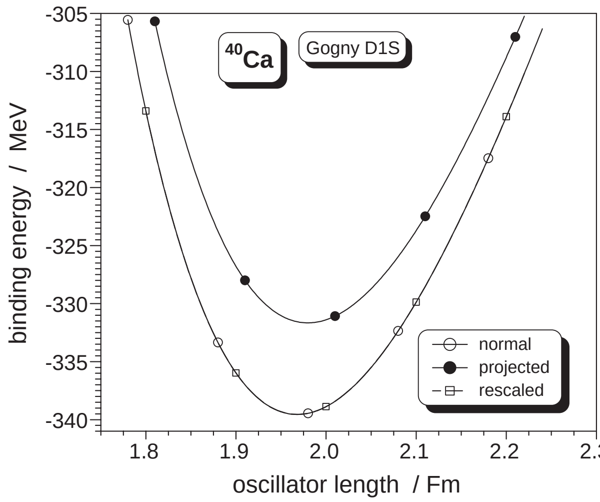

Fig. 10. The b-dependence of the total binding energy for ^{40}Ca as obtained with the D1S interaction in the normal approach (open circles) is compared to the result of the Galilei-invariant description (full circles). The open squares give the result, if in the projected approach the strength parameter t_0 of the density-dependent term is rescaled according to eq. (3.68) in order to compensate the difference in this term.

the partial cancellation of a rather large repulsive term and an even stronger central attraction. This is by the way rather similar as in the relativistic mean-field approaches and this may be at least one of the reasons why the Gogny interaction is so successfull.

Figure 7 gives the COM-projected results for the three doubly even nuclei, too. As expected, because of the additional repulsion of eq. (3.64) with respect to (3.63), by the projection into the COM rest frame we loose some binding energy and the minima are shifted (because of the $1/b^4$ dependence) to larger values of the oscillator length. At first one seems to observe a nice $1/A$ effect, however, one has to keep in mind that here energies per nucleon are plotted so that in order to obtain the differences in absolute binding energies the differences in fig. 7 have to be multiplied by the correponding mass number. This leads, *e.g.*, even in ^{40}Ca to a 10 MeV effect at the minimum as can be seen from fig. 10.

Figure 10 shows furthermore, that the (fitted) "normal" result can be reproduced by the COM-projected one by renormalizing the phenomenological constant t_0. Unfortunately, for this purpose the simple GOA

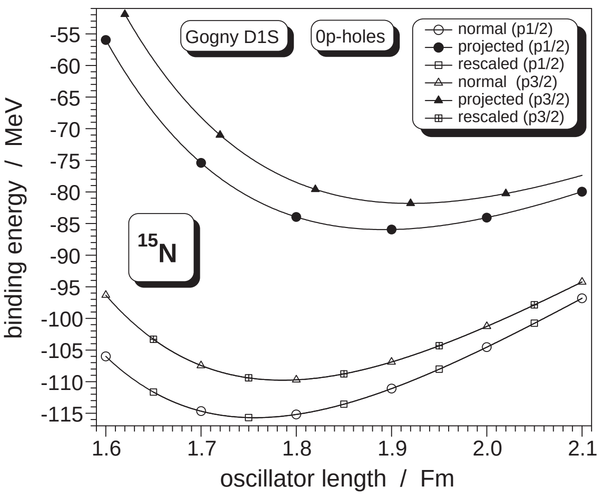

Fig. 11. The b-dependence of the total binding energies of the $0p$-proton holes in ^{16}O. Again we compare the normal, the projected and the rescaled results. The scaling factor compensating the difference in the density-dependent term is here given by eq. (3.69).

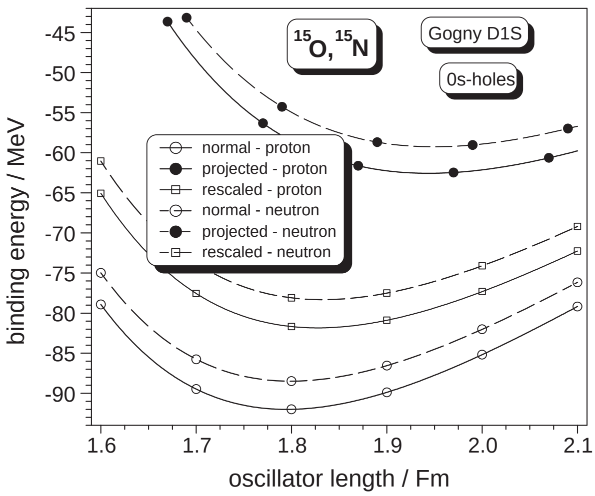

Fig. 12. Same as in fig. 11, but for the $0s$-holes in ^{16}O. Full lines refer to proton holes, the dashed ones to neutron holes. The rescaling is here given by eq. (3.70). The differences of the rescaled with respect to the normal results comes here from the density-independent parts of the Gogny interaction.

approach (3.67) is not sufficient. Instead, we have to use here

$$t_0(|\rangle, \text{proj}) = \frac{(A-8/7)\sqrt{A-8/7}}{(A-1)\sqrt{A}} \cdot \left\{1.00250862 + 0.36271335/A - 1.49099126/A^2\right\} t_0 \tag{3.68}$$

in order to reproduce the unprojected results in the three considered nuclei. It is easily checked that for $A = 4$ the polynomial part in (3.68) becomes unity.

In the one-hole case the effects of the COM projection are even larger. Figure 11 shows the $0p$-proton holes in ^{16}O with and without COM projection as functions of the oscillator length. Again, we have succeeded to reproduce the unprojected results by renormalizing the density-dependent term. However, here a reduction of t_0 according to (3.68) is not sufficient, but in order to compensate the COM effect in the density dependent for both the $0p$-holes in ^{16}O and ^{40}Ca now

$$t_0(b_{0ph}|\rangle, \text{proj}) = \frac{(A-8/7)\sqrt{A-8/7}}{(A-1)\sqrt{A}} \cdot \{1.03469739 - 1.28210705/A\}\ t_0 \tag{3.69}$$

has to be used.

Figure 12 displays the effects for the $0s$-holes in ^{16}O. For a compensation of the effects induced by the density-dependent term, here again a different rescaling of t_0 is needed:

$$t_0(b_{0sh}|\rangle, \text{proj}) = \frac{(A-8/7)\sqrt{A-8/7}}{(A-1)\sqrt{A}} \cdot \left\{1.04580859 - 1.42639837/A + 3.86691777/A^2\right\} t_0\,. \tag{3.70}$$

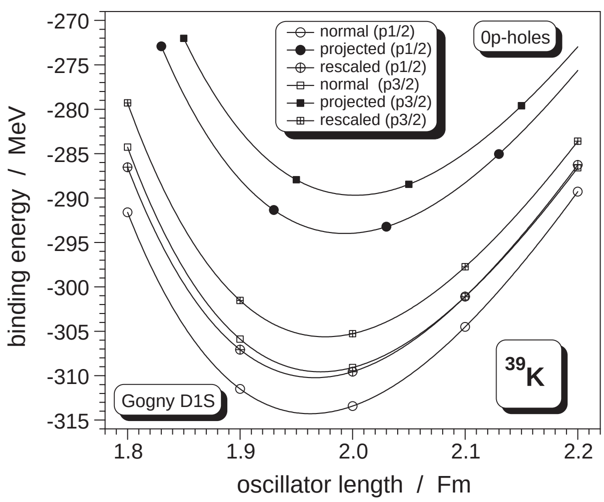

Fig. 13. Same as in fig. 11, but for the $0p$-proton holes in ^{40}Ca. The differences in the rescaled (eq. (3.69)) and the normal results comes here again from the density-independent parts of the interaction.

Note, that the remaining differences due to the density-independent terms are here much larger than in the B1 case discussed in sect. 2. This is due to the fact that the central component of the Gogny interaction is much stronger than that of the Brink-Boeker one.

Figure 13 displays the $0p$-proton holes in ^{40}Ca, fig. 14 the $0s$-holes in this nucleus. In both cases we have ensured by using (3.69) and (3.70), respectively, that the density-dependent term contributes with projection into the COM rest frame exactly the same amount as the unprojected normal approximation. Again it is seen that, because of the stronger central interaction, the remaining differences between projected and unprojected results which come from the density-independent parts of the interaction are larger in the Gogny-D1S than in the B1 case

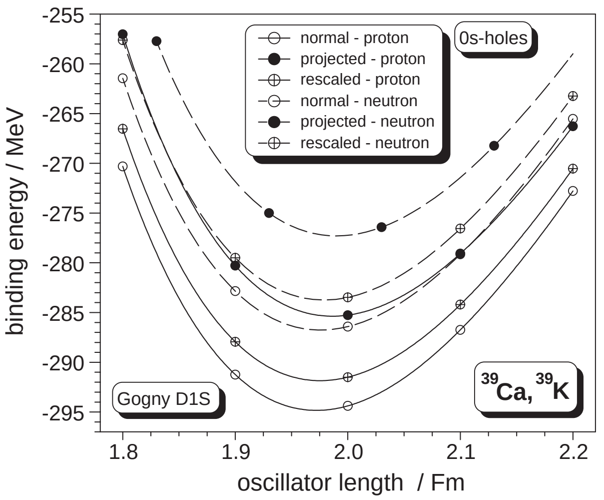

Fig. 14. Same as in fig. 12, but for the $0s$-holes in ^{40}Ca. Again the normal, the COM-projected and the rescaled results are compared with each other.

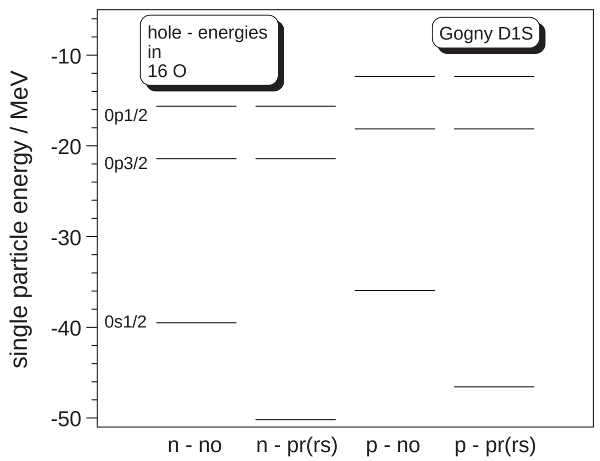

Fig. 15. The single-hole energies (2.64) of ^{16}O computed with the Gogny D1S interaction are displayed for fixed oscillator length b = 1.79 fm. The neutron-hole states are displayed in the first two, the proton-hole states in the last two columns. As in fig. 5 "no" refers to the normal, "pr" to the Galilei-invariant description.

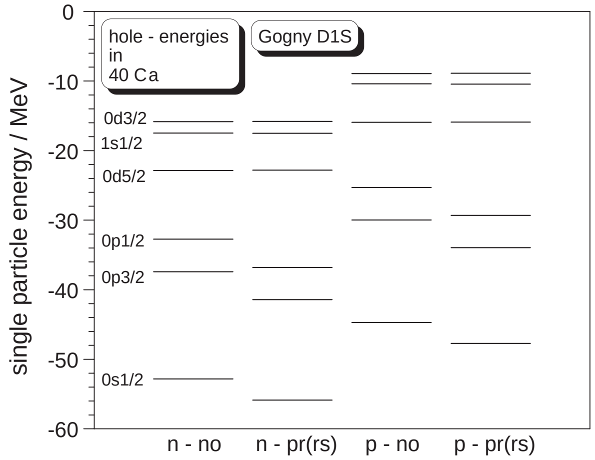

Fig. 16. Same as in fig. 15, but for the nucleus ^{40}Ca. Here a fixed oscillator length of b = 1.97 fm has been used.

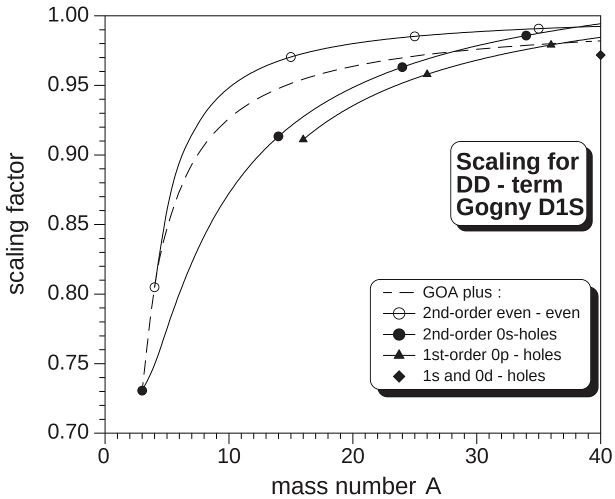

Fig. 17. The scaling factors compensating for the differences obtained for the density-dependent term of the Gogny D1S interaction in the COM projected with respect to the normal approach are plotted as functions of the mass number. The dashed line refers to the approximation (3.67), the open circles to (3.68), the full circles to (3.70) and the full triangles to (3.69). Finally, the full diamond gives the arithmetic average of the two values in (3.71).

discussed before. For the $1s$- and $0d$-holes in ^{40}Ca one has to use

$$\begin{aligned} t_0(b_{1sh}|\rangle, \text{proj}) &= \\ &(10.72996460769541/10.81454144450567)\, t_0\,, \\ t_0(|\, b_{0dh}|\rangle, \text{proj}) &= \\ &(10.79316541696027/10.87733257118675)\, t_0\,, \quad (3.71) \end{aligned}$$

in order to obtain identical results for the density-dependent term with and without the COM projection. The corresponding curves will not be shown in the present paper. Instead, we show in figs. 15 and 16 the single-hole energies computed according to eq. (2.64) with and without COM projection always at the oscillator lengths at the minima of fig. 7. For the projected case always the renormalized t_0 has been used. Obviously, the projection somewhat spoils the nice results for the deep-lying hole states, but this can be compensated by refitting also the density-independent term.

Figure 17 displays finally the formulas (3.67) to (3.71) used for renormalizing the parameter t_0. As can be seen, unfortunately these renormalisations are strongly state dependent and thus cannot be performed with some universal function like (3.67).

4 Conclusions

In the present paper we have investigated the effects of the restoration of Galilei invariance on the energies of simple oscillator configurations. Two different Hamiltonians have been studied. The first consists of the kinetic energy, the Coulomb interaction and a linear combination of central- and spin-orbit Gaussian interactions. The second has in addition a density-dependent term of the Gogny type. From both Hamiltonians the COM Hamiltonian has been subtracted in order to account for the trivial $1/A$ effect of the COM motion.

Let us start by summarizing the results for density-independent interactions. Provided that the COM Hamiltonian is subtracted, here the total Hamiltonian is translational invariant. For non-spurious oscillator configurations like the ground states of the three considered even-even nuclei ^{4}He, ^{16}O and ^{40}Ca as well as for the one-hole states out of the last occupied shell in these reference configurations thus the normal and the projected description give identical results, *i.e.* no effects of the restoration of Galilei invariance on top of the trivial $1/A$ effect are seen. The situation changes if holes with excitation energies $\geq 1\hbar\omega$ are considered. These configurations contain spurious admixtures due to the COM motion and thus the subtraction of the COM Hamiltonian alone is not sufficient. For the Brink-Boeker interaction B1 [8] on top of the usual $1/A$ correction, the full restoration of the Galilei invariance yields about 6 MeV for the $0p$-holes in ^{16}O, and for both the $0p$- as well as the $0s$-holes in ^{40}Ca still more than 2 MeV are obtained. These results nicely complement the results for the spectroscopic factors presented in ref. [1]. While in the normal approach they equal unity for all the occupied states, in the Galilei-invariant description the spurious admixtures in the deep-lying hole states are removed and thus a depletion of the occupation in these hole states is obtained. Since the total hole strength has nevertheless to be conserved, this depletion is compensated by an "over-occupation" of the last occupied shell. It is a rather satisfying result that the Kolthuns sum rule, obtained in the normal approach with the normal single-particle energies and spectroscopic factors, equal to unity, gives exactly the same result as the Galilei-invariant approach in which the COM-projected spectroscopic factors out of [1] and the COM-projected single-particle energies are used. This is a very good check of the consistency of the COM-projected description.

Obviously, as all density-independent effective interactions, the Brink-Boeker force B1 suffers from a serious deficiency: experimental total binding energies and root mean-square radii cannot be reproduced simultaneously, and the separation energies for nucleons out of deep-lying hole states are much larger than experimentally observed, a drawback which is even enlarged if Galilei invariance is respected. In order to cure these deficiencies usually density-dependent terms are introduced in the interaction. Examples are the zero-range Skyrme [10] and the finite-range Gogny interaction [6] which both have been used with great success in Hartree-Fock and Hartree-Fock-Bogoliubov calculations all over the nuclear mass table.

For such density-dependent interactions (we have used the Gogny-force D1S [9] as an example here), the restoration of Galilei invariance is considerably more complicated than for density-independent forces out of two reasons. First, instead of the "normal" density now the COM-projected one has to be used in the Hamiltonian and second, the momentum transfer connected to the dependence of the density of the COM coordinate of two nucleons has to be compensated by the other $A-2$ nucleons. This is achieved by a "recoil" operator which, unfortunately, makes out of the density-dependent term two-body interaction an A-body interaction.

The effects of the restoration of Galilei invariance for the energies of the simple configurations considered here are dramatic. Even for the (non-spurious) ground state of ^{40}Ca a difference of 10 MeV in the total binding energy is obtained with respect to the normal approximation and for the various hole states the effects are even larger. However, the Gogny force is purely phenomenological and its parameters have been adjusted doing calculations in the normal approximation. We have, therefore, rescaled the strength parameter of the density-dependent term in order to get the same results as in the normal approach at least for the non-spurious oscillator configurations. Unfortunately, this rescaling cannot be done with some universal function but is strongly mass and state dependent. Furthermore (because of the much stronger density-independent terms) the remaining energy differences for the deep-lying hole states obtained with the COM projected as compared to the normal approach become even larger than in the calculations with the Brink-Boeker interaction. In order to preserve the nice results of the Gogny interaction in the Galilei-invariant approach, thus, a careful refitting of not only the density-dependent but also the density-independent parts seems to be unavoidable.

Obviously, it would be interesting to investigate the effects of the restoration of the Galilei invariance also for more microscopic interactions like, *e.g.*, G-matrices obtained from realistic nucleon-nucleon interactions via solving the Bethe-Goldstone equation. This, however, requires a Galilei-invariant version of the latter, which up to now has not been derived, and goes beyond the scope of the present series of papers.

The author is grateful to Profs. L. Egido and L. Robledo from the Universidad Autonoma de Madrid for helpful discussions on the Gogny interaction.

References

1. K.W. Schmid, Eur. Phys. J. A **12**, 29 (2001), and references therein.
2. A.E.L. Dieperink, T. de Forest, Phys. Rev. C **10**, 543 (1974).
3. K.W. Schmid, Eur. Phys. J. A **13**, 319 (2002), and references therein.
4. L.J. Tassie, C.F. Barker, Phys. Rev. **111**, 940 (1958).
5. H. Müther, P.U. Sauer, in *Computational Nuclear Physics*, Vol. **2**, (Springer, New York, 1993) p. 30.
6. D. Gogny, *Nuclear Selfconsistent Fields*, edited by G. Ripka, M. Porneuf (North-Holland, Amsterdam, 1975).
7. D.J. Thouless, Nucl. Phys. **21**, 225 (1960).

8. D.M. Brink, E. Boeker, Nucl. Phys. A **91**, 1 (1966).
9. J.F. Berger, M. Girod, D. Gogny, Comp. Phys. Commun. **63**, 365 (1991).
10. T.H.R. Skyrme, Philos. Mag. **1**, 1 (1956); Nucl. Phys. **9**, 615 (1959).